KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 312

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION I

STREAM: SESSION III

DAY: THURSDAY

TIME: 9.00 – 11.00 A.M.

DATE: 12/08/2010

INSTRUCTIONS:

Answer Question ONE and any other TWO Questions

PLEASE TURNOVER

QUESTION ONE: 30 MARKS

a). Define the following terms as used in ordinary differential equations; differential equation, order, linearity and homogeneous. (8 marks)

b). Test whether the following differential equation are exact and hence solve (x+2y)dx + (2x +y)dy = 0 (5 marks)

c). The population of a country increases at a rate proportional to the current population. If the population doubles in 40 years, in how many years will it triple? (8 marks)

d). Show that the equation $x^3y''' - 6xy' + 12y = 0$ has three linearly independent solutions each of the form $y = x^r$ (4 marks)

e). Solve the initial value problem $(1 + y^2)dx = (1 + x^2)dy = 0$

With the initial conditions that when $x = 0, y = 1$. (5 marks)

QUESTION TWO: 20 MARKS

a). Show that the equation $\{\cos(t + x^2) + 3x\}dt + \{2x\cos(t + x^2) + 3t\}dx = 0$ is an exact equation hence find its solution. (6 marks)

b). Show that $f_1(x) = \cos x$ and $f_2(x) = \sin x$ are linearly independent solutions of the differential equation $y'' + y = 0$ (4 marks)

c). Eliminate the constants to obtain the general equation whose general solution is

$y = c_1x^2 + c_2e^{2x}$ where c_1 and c_2 are arbitrary constants. (5 marks)

d). By using a suitable integrating factor solve

$(3x^4y - 1)dx + x^5dy = 0$ when $x = 1$, and $y = 1$. (5 marks)

QUESTION THREE: 20 MARKS

a). Find the solution of the following initial value problems of the second order

$$y'' - 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = -1.$$ (5 marks)

b). The sum of Ksh 5,000 is invested in a bank which pays interest at a rate of 8% per annum compounded continuously. The following separable equation describes the amount of money at any time t. $\frac{dy}{dx} = \frac{8}{100}y$

Find the amount of money after 25 years. (6 marks)

c). Show that equation y = $Ae^{2x} + Be^{-x}$ is the solution of the differential equation defined by;

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$ (4 marks)

d). Consider the differential equation $y'' + 4y = 0$. Determine whether $y(x) = k_1 \sin 2x + k_2 \cos 2x$ is its solution where k_1 and k_2 are arbitrary constants.

(5 marks)

QUESTION FOUR: 20 MARKS

a). Determine c_1 and c_2 so that $y(x) = c_1e^{2x} + c_2e^{x} + 2\sin x$ will satisfy the conditions $y(0) = 0$ and $y'(0) = 0$. (6 marks)

b). Solve $\left(D^2 - 4\right)y = x^2$. (4 marks)

c). Given the differential equation $\left(D^2 - 7D + 10\right)y = e^{2x} + e^{5x}$. Determine the complementary function and the particular integral, hence write the general solution.

(6 marks)

d). Use the method of separation of variables to solve the differential equation:
$\frac{dy}{dx} = \left(1 + y^2\right)e^{x}$. (4 marks)

QUESTION FIVE: 20 MARKS

a). Use the method of undetermined coefficient to find P.I of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 2x - 40\cos 2x$$ (8 marks)

b). Solve the differential equation $(xy+1)dx + x(x+4y-2)dy = 0$ (6 marks)

c). The following equation is related to biophysical limitations in the study of deep diving $y' = Ay + B + be^{-ax}$. Show that the solution is of the form $y = \frac{-B}{A} - \frac{b}{(a+A)}e^{-ax} + ce^{Ax}$ where c is an arbitrary constant. (6 marks)