KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 410**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATION I**

STREAM: **Y4S1**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **02/12/2009**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Show that the equation;

$$x = u + v$$
$$y = u - v$$
$$z = 4uv$$

represents a surface hence find its constraint form. **(5 marks)**

(b) Find the Equation of the Tangent plane to the surface $4x^2 - 9y^2 - 9z^2 - 36 = 0$ at the point $[3\sqrt{3},\ 2,\ 2]$ **(5 marks)**

(c) Solve;

$$\frac{dx}{x+y} = \frac{dy}{y} = \frac{dz}{z+y^2}$$ **(5 marks)**

(d) Find an Integrating factor for the Equation $(y + x^2y^2)dx = xdy$ and solve it. **(5 marks)**

(e) Verify that the differential equation $(y^2 + zy)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and find its solution. **(5 marks)**

(f) Form a quasi-linear partial differential equation of 1st order whose solution is $\emptyset(x^2 e^z \quad ye^z) = 0$ **(5 marks)**

QUESTION TWO (20 MARKS)

(a) A surface is defined by;

$$x = u; \qquad y = v; \qquad z = \frac{1}{4}(u^2 - v^2)$$

Find (i) The Equation of the Tangent plane

(ii) The Equation of the normal line at the point $(3, 1, 2)$ **(10 marks)**

(b) Find the Equation of

(i) The tangent line

(ii) The Normal plane

To the curve; $x = t - \cos t$

$y = 3 - \sin 2t$

$z = 1 - \cos 3t$ at the point $t = \frac{\pi}{2}$ **(10 marks)**

QUESTION THREE (20 MARKS)

(a) Reduce the simultaneous ordinary differential equation defined by,

$$P_1\,dx + Q_1 dy + R_1 dz = 0$$

$$P_2\,dx + Q_2\,dy + R_2\,dz = 0$$

To a single ordinary differential equation, hence solve

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$$ **(10 marks)**

(b) Find the orthogonal trajectories on the cone. $x^2 + y^2 = z^2 \tan^2\alpha$ of its intersections with the family of planes parallel to $z = 0$ **(10 marks)**

QUESTION FOUR (20 MARKS)

(a) Find an integrating factor and solve the equation defined by

$$2x^2y\,dx + (x^3 + 2xy)\,dy = 0$$ **(10 marks)**

(c) The acceleration of a particle moving in a straight line is negative of its velocity. If it starts from the origin with velocity equal to one, find its position at the end of two units of time. **(10 marks)**

QUESTION FIVE (20 MARKS)

(a) Verify that the differential equation; $y\,dx - (x+z)dy + ydz = 0$ is homogeneous hence find its primitive. **(6 marks)**

(b) Form a potential differential equation whose solution is

$$\emptyset\,(x^2 + y^2 + z^2, xyz) = 0$$ **(7 marks)**

(c) Find the general solution of; $xzx + z\,2y = y$ **(7 marks)**