KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 312**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATION I**

STREAM: **SESSION VII, VIII & IX**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **12/08/2009**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions.

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Classify the differential equations as to order, degree and linearity.

(i) $3x^2 \frac{d^3y}{dx^3} - \sin x \frac{d^2y}{dx^2} - \cos xy = 0$

(ii) $7(y'')^3 + 2y' + 2 \times y^2 = 0$ **(3 marks)**

(b) Find the differential associated with the primitive

$$y = Ae^{2x} + Be^{x} + c$$ **(5 marks)**

(c) Show that the differential equation $3x^2y\,dx + (x^3 - y^2)dy = 0$ is exact and hence or otherwise solve it. **(4 marks)**

(d) A radio active isotope remains unused in a laboratory for 10 years after which it is found to contain only 80% of the original mass. Find

(i) The half of the isotope

(ii) How many years it will take until only 15% of the original mass is left. **(6 marks)**

(e) Find the nature of the roots of the auxillary equations of the given differential equations and hence solve them

(i) $(3D^3 - 2D^2 - D)y = 0$ **(4 marks)**

(ii) $y'' + y = 0$ **(4 marks)**

QUESTION TWO (20 MARKS)

(a) Solve the differential equation

$$x^2 \frac{dy}{dx} + 3xy = 1$$ **(4 marks)**

(b) Show that $\frac{1}{x^2}$ is an integrating factor of the differential equation.
$(3x^2 + y^2)dx - 2xy\,dy = 0$ and solve it. **(6 marks)**

(c) Find the general solution of the differential equation

$\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + 3\frac{d^3y}{dx^2} - 10\frac{dy}{dx} + 18 = 0$ y one root is 1 + i and complex roots occurs in conjugate pairs. **(10 marks)**

QUESTION THREE (20 MARKS)

(a) Use the substitute $y = vx$ to solve the equation $x(x-y)\frac{dy}{dx} + y^2 = 0$ **(6 marks)**

(b) Use the method of undetermined coefficients to solve the differential equation
$y'' - y' - 2y = \sin x$ **(7 marks)**

(c) Solve the equation $\frac{d^2y}{dx^2} + y = \csc x$ using the method of variation of parameters. **(7 marks)**

QUESTION FOUR (20 MARKS)

(a) Find the power series solution of the equation $(x^2 - 1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0,$

$y(0) = 4$ $y'(0) = 6$ by Taylor's series expansion method. **(10 marks)**

(b) Find the solution of the homogeneous system. **(6 marks)**

$$\frac{dx}{dt} - 3x + 18y = 0$$
$$\frac{dy}{dt} - 2x + 9y = 0$$ **(10 marks)**

QUESTION FIVE (20 MARKS)

(a) If a and b are arbitrary constants find the second order differential equation whose solutions is $y = ax + \frac{b}{x}$ **(4 marks)**

(b) Prove that the transformation $V = y^{1-n}$ reduces the equation $\frac{dy}{dx} + p(x)y = Q(x)y^n$ to a linear equation in V and x. Hence solve the initial value problem. **(10 marks)**

$\frac{dy}{dx} + \frac{x}{2x} = \frac{x}{y^3}$ $y(1) - 2$

(c) Find the solution of the differential equation

$\sin x\frac{dy}{dx} - y\cos x = \sin^2 x \cos x$ given that $y = 2$ when $x = {}^{\pi}/_{2}$. **(6 marks)**