KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 322**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATION II**

STREAM: **SESSION VI**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **12/08/2009**

INSTRUCTIONS:

Attempt **Question ONE** and **Any other TWO**

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

a) Obtain two linearly independent solutions of the differential equations:

$$2xy^{11} + 5(1 - 2x)\, y^{1} - 5y = 0$$

Valid near the origin for x>0 using power series method (10 mks)

b) Use elementary eliminations calculus to solve :

$Y^1 = 2Y + Z$

$Z^1 = - 4Y + 2Z$ (6 mks)

c) Write the following differential equations as the first order system:

i) $Y^{11} + 4y^1 + 4y = e^x$ (2mks)

ii) $Y^{111} + py^{11} + qy^1 + ry = f(x)$ (3mks)

d) Identify and classify all singular base of the following differential equations:

i) $x(x-1)^2\,(x+2)y^{11} + x^2y^1 - (x^3 + 2x - 1)\, y = 0$

(3mks)

ii) $x^4(x^2 + 1)\,(x-y)^2y^{11} + 4x^3(x-1)y^1 + (x+1)\, y = 0$

(3mks)

e) Solve the following system:

$$X^1 = AX \text{ for } A = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$$

(3mks)

QUESTIONTWO (20 MARKS)

a) Solve the system $x^1 = Ax + B$ for $A = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}$

$$B = \begin{bmatrix} 3 & e^{2t} \\ t & e^{2t} \end{bmatrix}$$

b) Consider the Richardson model

$$\frac{dx}{dt} = ay - px + r$$

$$\frac{dy}{dt} = bx - qy + s$$

Where a, b, p and q are positive constants r and s having any value.
This model is used to study the arms raises for two countries with x and y expenditures for armaments respectively .Investigate this model for a = 2, b = 4, p = 5, q = 3, r =1, s = 2, $x_0 = 8$, $y_0 = 7$

(10 mks)

QUESTION THREE (20MARKS)

a) Solve the system:

X^1=AX for A= $\begin{pmatrix} 0 & 1 \\ -4 & 4 \end{pmatrix}$ (10 mks)

b) Find a solution of the initial value problem:

$x^1 = x^2$, $x_0 = 0$, $y_0 = 1$

Using Picard iterates define below:

$y_0(x) = y_0$

$y1 = yo + \int_0^x f(x, yo(t))$ dt

1 1

1 1

1 1

$y_n = y_0 + \int_0^x f(x,\ y_{n-1}\ (t))dt$ (10 mks)

QUESTION FOUR (20 MARKS)

a) Two coils of a transformer are identical with resistance R, inductance L, mutual inductance M. A Voltage E is impressed on the primary coil. Determine the current at any instant assuming that there is no currents in either coil initially

(10 mks)

b) Solve the following first order non homogenous system:

X1 = Ax + f(t)

For A = $\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$ $and\ f(x) = \begin{pmatrix} f, & (t) \\ f_2 & (t) \end{pmatrix}$

(10 mks)

QUESTION FIVE (10 MKS)

a) Identify and classify all singular points of:

i) $x^3(x-1)\,y^{11} + (x-1)y^1 + 4xy = 0$ (3mks)

ii) $xy^{11} + y = 0$ (2mks)

b) Write the following differential equations as the first order system:

i) $Y^{11} + zy^{11} - 8y = 0$ (3mks)

ii) $Y^{11} + x = 3$ (2mks)

c) Solve the system:

$X^1 = AX$ for $A = \begin{bmatrix} 4 & 1 \\ -4 & 8 \end{bmatrix}$ (5mks)

d) Use elementary elimination calculus to solve the following system of first order deferential equation:

$Z^1 = 4\,Z - W$

$W^1 = -\,4Z + 4W$ (5mks)