KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 322**

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION II

STREAM: **SESSION VI**

DAY: **THURSDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **27/11/2008**

INSTRUCTIONS:

Answer Question ONE and any other TWO

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Obtain two linearly independent solution of the differential equation

$$4xy^{11} + 2y^{1} + y = 0$$

valid near the origin for $x > 0$ using power series method **(10 marks)**

(b) Identify and classify all singular points of:

i. $4xy^{11} + 2y^{1} + y = 0$ **(2 marks)**

ii. $9x(1-x)y^{11} - 12y^{1} + 4y = 0$ **(3 marks)**

iii. $(x - x^2)y^{11} + (1 - x)y^{1} - y = 0$ **(3 marks)**

(c) Use elementary elimination calculus to solve the following system of first order differential equation:

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -2x + 3y$$ **(6 marks)**

(d) Write the following differential equations as a first order system:

i. $y^{11} + py^{1} + qy = f(x)$ **(2 marks)**

ii. $y^{111} + py^{11} + qy^{1} + ry = f(x)$ **(4 marks)**

QUESTION TWO (20 MARKS)

(a) Fine two linear independent solutions to the system;

$$x^{1} = Ax \text{ for } A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

(8 marks)

(b) Solve the system;

$$x^{1} = Ax \text{ for } A = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix}$$

(12 marks)

QUESTION THREE (20 MARKS).

(a) Solve the system ;

$$X^1 = Ax + f(t) \text{ for } A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \text{ and } f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

(10 marks)

(b) Two coils of a transformer are identical with resistance R, inductance L, mutual inductance M. A voltage E is impressed on the primary coil.

Determine the currents in the coils at any instant assuming that there is no current in either coil initially. **(10 marks)**

QUESTION FOUR (20 MARKS)

(a) Consider the following sequence of functions

$$y_0(x) = y_0$$

$$y_1 = y_0 + \int_0^x f\big(t, y_0(t)\big)dx$$

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$$y_n(x) = y_0 + \int_0^x f\big(t, y_{n-1}(t)\big)dt$$

Show that the above sequence converges to a solution for the initial value problem

$y^1 = x^2$, $x_0 = 0$, $y_0 = 1$ **(10 marks)**

(b) Consider the Richardson model

$$\frac{dx}{dt} = ay - px + r$$

$$\frac{dy}{dx} = bx - qy + s$$

Where a, b. p and q are positive constants r and s having any value.

This model is used to study the arms races for two countries with x and y expenditures for armaments respectively.

Investigate this model for a = 4, b = 2, p =3, q= 1, r = 2, s = 2, x_0 = 4 and y_0= 1 **(10 marks)**

QUESTION FIVE (20 MARKS)

(a) Solve the system

$x^1 = 2x - 5y$

$y^1 = 2x - 4y$ **(12 marks)**

(b) Test whether the solution to the system in (a) above is linearly independent at

$t = t_0 = 0$ **(4 marks)**

(c) Reduce the following differential equation into a first order system:

$y^{1V} - y = 0$ **(4 marks)**