KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 322

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION II

STREAM: SESSION VI & VIII

DAY: SATURDAY

TIME: 9.00 – 11.00 A.M.

DATE: 28/11/2009

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE

(a) Write the following differential equations as a first order system:

(i) $y^{11} - 6y^{1} + 8y = x + z$ **(2 marks)**

(ii) $y^{v} - y = 0$ **(4 marks)**

(b) Use elementary elimination calculus to solve the following systems of equation:

$$y^{1} = 2y + z$$
$$z^{1} = -4y + 2z$$
(8 marks)

(c) Identify and classify all singular points of the following differential equations:

(i) $x^3(x-1)y^{11} + (x-1)y^{1} + 4xy = 0$ **(3 marks)**

(ii) $(x^2 + 1)(x-4)^3 y^{11} + (x-4)^2 y^{1} + y = 0$ **(3 marks)**

(d) Obtain two linearly independent solution of the differential equation

$$2xy^{11} + (1+x)y^{1} - 2y = 0$$

valid near the origin for $x > 0$ using power series method. **(10 marks)**

QUESTION TWO

(a) Find a linearly independent solution to the system $x^{1} = AX$ for $A = \begin{pmatrix} 8 & -1 \\ 4 & 12 \end{pmatrix}$

(10 marks)

(b) Find a solution of the initial value problem

$$y^{1} = x^2, \quad x_0 = 0, \quad y_0 = 1$$

Using Picard iterate define as

$$y_0(1) = y_0$$
$$y_1 = y_0 + \int_0^x f(x, y_0(t))\, dt$$
$$! \qquad ! \qquad\qquad !$$
$$! \qquad ! \qquad\qquad !$$
$$y_n = y_0 + \int_0^x f(x, y_{n-1}(t))\, dt$$
(10 marks)

QUESTION THREE

(a) Two coils of a transformer are identical with resistance R, inductance m, mutual inductance. A voltage E is impressed on the primary coil. Determine the currents in the coils at any instant t assuming that there is no current in either coils initially. **(10 marks)**

(b) Consider the Richardson model

$$\frac{dx}{dt} = ay - px + r$$

$$\frac{dy}{dx} = bx - qy + s$$

Where a, b, p, and q are positive constants, rad s caring any value. This model is used to study the areas races for two ar respectively. Investigate this model for $a = 4,\ b = 2,\ p = 3,\ q = 1,\ r = 1,\ r = 2,\ S = 2\ \ x_0 = 4,\ and\ y_0 = 1$

(10 marks)

QUESTION FOUR

(a) Solve the system

$$x^1 = 2x - 5y$$

$$y^1 = 2x - 4y$$

(10 marks)

(b) Solve the following non homogeneous system:

$x^1 = AX + B$ for A = $\begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}, B\ \begin{pmatrix} 3e^{2t} \\ te^{2t} \end{pmatrix}$ **(10 marks)**

QUESTION FIVE

(a) Reduce the following into a first order system:

$y^{11} + 4y^1 + 4y = e^x$ **(2 marks)**

(b) Identify and classify all singular points of

(i) $x^3(x^2 - 4)y^{11} + 2(x^2 - 4)y^1 - xy = 0$ **(3 marks)**

(ii) $(x^2 + 6x + 8)y^{11} + 3y = 0$ **(2 marks)**

(c) For a linear differential equation $b(x)y^n + b_1(x)y^{n-1} + ----- + b_n(x)y = R(x)$ what you understand by the term

(i) Ordinary point of the differential equation **(1 mark)**

(ii) Regular singular point of the differential equation. **(2 marks)**

(iii) Irregular singular point of the differential equation. **(1 mark)**

(d) Use power series method to solve the following differential equation

$x^2y^{11} + 3xy^1 + (1-2x)y = 0$ valid for $x > 0$ **(9 marks)**