KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 322**

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION II

STREAM: **SESSION VI & VIII**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 P.M.**

DATE: **08/04/2009**

INSTRUCTIONS:

1. Question ONE is compulsory.
2. Attempt question ONE and any other TWO

PLEASE TURN OVER

QUESTION ONE-COMPULSORY (30 marks)

(a) Identify and classify all singular points of

(i) $x(x-1)^2(x+2)y^{11} + x^2 y^1 - (x^3 + 2x - 1)y = 0$ **(3 marks)**

(ii) $x^4(x^2+1)(x-1)^2 y^{11} + 4x^3(x-1)y^1 + (x+1)y = 0$ **(3 marks)**

(b) Use elementary elimination calculus to solve the following system of first order differential equation

$$u^1 = 4u - v$$

$$v^1 = 4u + 4v$$ **(8 marks)**

(c) Write the following differential equation as first order system:

(i) $y^{111} + py^{11} + qy^1 + ry = f(x)$ **(3 marks)**

(ii) $y^{1v} - y = 0$ **(3 marks)**

(d) Obtain two linearly independent solution of the differential equation

$$4xy^{11} + 3y^1 + 3y = 0$$

Valid near the origins for $x > 0$ using power series method **(10 marks)**

QUESTION TWO (20 MARKS)

(a) Find a linear independent solution to the system

$$x^1 = Ax \quad \text{for } A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$ **(10 marks)**

(b) Solve the system

$x^1 = 2x - 5y$

$y^1 = 2x - 4y$ **(10 marks)**

QUESTION THREE (20 Marks)

(a) Two coils of a transformer are identical with resistance R, inductance L, mutual inductance M. A voltage E is impressed on the primary coil. Determine the currents in the coils at any instant assuming that there is no current in either coil initially. **(10 marks)**

(b) Solve the system

$$X^1 = Ax + B \quad \text{for} \qquad A = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 3 & e^{2t} \\ t & e^{2t} \end{pmatrix}$$ **(10 marks)**

QUESTION FOUR (20 Marks)

(a) Consider the Richardson model

$$\frac{dx}{dt} = ay - px + r$$

$$\frac{dy}{dx} = bx - qy + s$$

Where a, b. p and q are positive constants r and s having any value.

This model is used to study the arms races for two countries with x and y expenditures for armaments respectively.

Investigate this model for a = 4, b = 2, p =3, q= 1, r = 2, s = 2, x_0 = 4 and y_0= 1 **(10 marks)**

(b) Find a linearly independent solution to the system:

$$x^1 = Ax \qquad \text{for} \qquad A = \begin{pmatrix} 4 & 1 \\ -8 & 8 \end{pmatrix}$$ **(10 marks)**

QUESTION FIVE (20 Marks)

(a) Use elementary elimination calculus to solve the following first order system:

$$y^1 = 2y + z$$

$$z^1 = -4y + 2z$$

(10 marks)

(b) Solve the system

$$x^1 = Ax + f(t) \qquad \text{for} \qquad A = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}$$

$$f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

(10 marks)