KABARAK   **UNIVERSITY**

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 312**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATIONS**

STREAM: **Y3S1**

DAY: **WEDNESDAY**

TIME: **2.00-4.00 P.M.**

DATE: **10/12/2008**

INSTRUCTIONS:

Answer question ONE and any other TWO questions.

PLEASE TURN OVER

Question One (30 Marks)

a) Consider the differential equation $y'' + 4y = 0$. Determine whether

$y(x) = k_1 \sin 2x + k_2 \cos 2x$ is its solution where k_1 and k_2 are arbitrary constants.

(4 Marks)

b) Determine c_1 and c_2 so that $y(x) = c_1 e^{2x} + c_2 e^x + 2\sin x$ will satisfy the conditions

$y(0) = 0$ and $y'(0) = 0$. **(5 Marks)**

c) Solve $(D^2 - 4)y = x^2$. **(6 Marks)**

d) For the equation $\dfrac{d^2 y}{dx^2} + 9y = 0$, show that $y_1 = \cos 3x$ and $y_2 = \sin 3x$ are two linearly

independent solutions. **(3 Marks)**

e) The differential equation for a circuit in which self-inductance and capacitance

neutralize each other is $L\dfrac{d^2 I}{dt^2} + \dfrac{I}{C} = 0$. Find the current I as a function of t given that

I is the maximum current and $i = 0$ when $t = 0$. **(6 Marks)**

f) Solve $\left(x^4 - 2xy^2 + y^4\right)dx - \left(2x^2 y - 4xy^3 + \sin y\right)dy = 0$. (6 Marks)

Question Two (20 Marks)

a) Solve the following homogeneous differential equation $\dfrac{dy}{dx} = 5\dfrac{y}{x} + 3\dfrac{x}{y}$. **(8 Marks)**

b) Solve the simultaneously equations,

$$\frac{dx}{dt} + wy = 0] \;] \;] \;] \;] \; (i)$$
$$\frac{dy}{dt} - wx = 0] \;] \;] \;] \;] \; (ii)$$

(6 Marks)

c) Solve the initial value problem: $dy = x\sqrt{x^2 + 1}\,dx$, $y(0) = 1$. **(6 Marks)**

Question Three (**20 Marks**)

a) Show that the differential equation $\left(3x^2 y^4 + 2xy\right)dx + \left(2x^3 y^3 - x^3\right)dy = 0$ is not exact.

Then reduce it to an exact form and solve it. **(12 Marks)**

b) Given the differential equation $\left(D^2 - 7D + 10\right)y = e^{2x} + e^{5x}$. Determine the

complementary function and the particular integral, hence write the general solution.

(8 Marks)

Question Four (**20 Marks**)

a) The population of a country increases at the rate proportional to the current population.

If the population doubles in 40 years, in how many years will it triple itself. **(10 Marks)**

b) Solve the differential equation $\dfrac{d^2 y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$, subject to $y'(0) = -1$ and

$y(0) = 1$. **(10 Marks)**

Question Five (**20 Marks**)

a) David deposits Ksh.200,000 into an account in which interest accumulates at the rate of 5% per year, compounded annually. He plans to withdraw Ksh.30,000 per year.

  (i) Set up and solve a differential equation to determine the value $Q(t)$ of his account

   t years after the initial deposit. **(10 Marks)**

  (ii) How long does it take for his account to be exhausted? **(5 Marks)**

b) Use the method of separation of variables to solve the differential equation:

$\dfrac{dy}{dx} = \left(1 + y^2\right)e^x$. **(5 Marks)**