KABARAK   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 312

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION I

STREAM: SESSION III

DAY: FRIDAY

TIME: 2.00 – 4.00 P.M.

DATE: 26/11/2010

INSTRUCTIONS:

1. Question ONE is compulsory.

2. Attempt question ONE and any other TWO

PLEASE TURNOVER

QUESTION ONE: 30 MARKS

a). Define the following terms as used in ordinary differential equations; differential equation,

order, linearity and homogeneous. [8 marks]

b). Solve the initial value problem $(1 + y^2)dx = (1 + x^2)dy = 0$

With the initial conditions that when $x = 0, y = 1$. [4 marks]

c). Find the orthogonal trajectories of all parabolas with vertices at the origin and foci on the x-axis

[4 marks]

d). Show that the equation $x^3 y''' - 6xy' + 12y = 0$ has three linearly independent solutions each

of the form $y = x^r$ [8 marks]

e). Determine the solution of the following non-homogeneous equation by the method of variation

of parameters $\quad \frac{d^2 y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1 + e^{-x}}$ [6 marks]

QUESTION TWO: 20 MARKS

a). A large tank contains 81 gallons of brine in which 20 pounds of salt is dissolved. Brine containing 3

pounds of dissolved salt per gallon runs into the tank at a rate of 5 gallons per minute. The mixture

is kept uniform by stirring, runs out of the tank at a rate of 2 gallons per minute. How much salt is

in the tank at the end of 37 minutes. [10 marks]

b). Define what is an exact differential equation. Show that the equation

$\{cos(t + x^2) + 3x\}dt + \{2xcos(t + x^2) + 3t\}dx = 0$

Is an exact equation hence find its solution. [10 marks]

QUESTION THREE: 20 MARKS

a). The following equation is related to biophysical limitations in the study of deep diving

$y' = AY + B + Be^{-ax}$ where a, b, A and B are constants show that the general solution of

this equation is given by

$$y' = -\frac{B}{A} - \frac{b}{a+A}e^{-ax} + ce^{Ax}$$

Where c is an arbitrary constant. [8 marks]

b). Given that $f_1(x) = x^2$ and $f_2(x) = cosx$, find the Wronskian of $f_1(x)$ and $f_2(x)$. [4 marks]

c). Show that $f_1(x) = cosx$ and $f_2(x) = sinx$ are linearly independent solutions of the differential

equation $y'' + y = 0$ [8 marks]

QUESTION FOUR: 20 MARKS

a). Find the solution of the following initial value problems of the second order

$$y'' - 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = -1.$$ [10 marks]

b). The sum of Ksh 5,000 is invested in a bank which pays interest at a rate of 8% per annum

compounded continuously. The following separable equation describes the amount of money

at any time t.

$$\frac{dy}{dx} = \frac{8}{100}y$$

Find the total amount of money after 25 years. [10 marks]

QUESTION FIVE: 20 MARKS

a). By using suitable transformation to reduce the equation to a separable equation, solve

$$(x + y)dx + (3x + 3y - 4)dy = 0$$

[8 marks]

b). Eliminate the constants to obtain the general equation whose general solution is

$y = c_1 x^2 + c_2 e^{2x}$ where c_1 and c_2 are arbitrary constants.

[6 marks]

c). By using a suitable integrating factor solve

$(3x^4 y - 1)dx + x^5 dy = 0$ when $x = 1$, and $y = 1$.

[6 marks]