KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 322

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION II

STREAM: SESSION VI

DAY: FRIDAY

TIME: 2.00 – 4.00 P.M.

DATE: 26/11/2010

INSTRUCTIONS:

- **Answer Question ONE and any other TWO Questions**

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

a) Obtain two linearly independent solution of the differential equation $4xy'' + 2y' + y = 0$ valid near the origin for x > 0 using power series method **(10 marks)**

b) Show that $e^{ax}, xe^{ax}, x^2e^{ax}$ are solutions of the differential equation

$\frac{d^3y}{dx^3} - 3a\frac{d^2y}{dx^2} + 3a^2\frac{dy}{dx} - a^3y = 0$ hence find the general solution of this equation.

(6marks)

c) Use elementary elimination calculus to solve the following system of first order differential equation: $\begin{aligned} x' &= 3x - 4y \\ y' &= 4x - 7y \end{aligned}$ where $(')$ denote differentiation with respect to time t.

(6 marks)

d) By use of a suitable integrating factor solve the differential equation

$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ **(8 marks)**

QUESTION TWO (20 MARKS)

a) Find the general solution of the system

$$\begin{aligned} &D^2y + (D-1)v = 0 \\ &(2D-1)y + (D-1)w = 0 \\ &(D+3)y + (D-4)v + 3w = 0 \end{aligned}$$

(10 marks)

b) Check that $(x(t),\ y(t)) = (e^{-4t} - 3e^{-2t},\quad -3e^{-4t} + 3e^{-2t})$ is a solution of the system of differential equations below and test for their linear independence. **(6 marks)**

$$\begin{aligned} x' &= -x + y \\ y' &= -3x - 5y \end{aligned}$$

c) Reduce the following differential equation into a first order system:

$y^{(iv)} - y = 0$ **(4 marks)**

QUESTION THREE (20 MARKS)

(a) Solve the uncoupled system

$$x' = -x - 3y$$
$$y' = 2y$$

(10 marks)

(b). Find the solution of the Legendre equation $(1-x^2)y'' - 2xy' + 2y = 0, \quad -1 < x < 1.$ using reduction of order technique using the fact that $\phi_1(x) = x$ is a known solution. **(10 marks)**

QUESTION FOUR (20 MARKS)

(a) Consider the system $\dot{X} = AX$ where $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $A = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$ is the uncoupled matrix of coefficients. Find the solution of the system and sketch the phase portrait showing direction of flow of vectors. **(8 marks)**

(b) Use the method of undetermined coefficients to solve the equation

$y'' + y' - 2y = 2x - 40\cos 2x$ **(7 marks)**

(c) Find the equilibrium points of the Duffing system $\begin{aligned} x' &= y \\ y' &= -x + x^3 - y \end{aligned}$ **(5 marks)**

QUESTION FIVE (20 MARKS)

(a) Determine the solution of the following non-homogeneous equation by the method of variation of parameters $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = \dfrac{1}{1+e^{-x}}$ **(8 marks)**

(b) The vertical motion y_1 and y_2 of a mechanical system comprising of two coupled springs with spring constants k_1 and k_2, with masses m_1 and m_2 suspended is governed by the simultaneous differential equations $\begin{aligned} m_1\ddot{y}_1 &= -k_1 y_1 + k_2(y_2 - y_1) \\ m_2\ddot{y}_2 &= -k_2(y_2 - y_1) \end{aligned}$

Using $m_1 = m_2 = 1$, $k_1 = 3$, $k_2 = 2$, find the equation governing the motion of the coupled springs when vibrating. **(12 marks)**