KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 312

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS

STREAM: Y3S1

DAY: WEDNESDAY

TIME: 2.00 – 4.00 P.M.

DATE: 23/03/2011

INSTRUCTIONS:

1.Question **ONE** is compulsory.

2. Attempt question ONE and any other TWO

PLEASE TURN OVER

QUESTION ONE

(i) Obtain a differential equation associated with the primitive $y = Ae^{2x} + Be^{x} + C$ and state its order (8mks)

(ii) Solve $ye^{x^2}dx + \frac{y^3 - 1}{x}dy = 0$ (5mks)

(iii) In a certain culture of bacteria, the rate of increase is proportional to the number present.

(a) If it is found that the number doubles in 4 hours, how many may be expected at the end of 12 hours? 9mks)

(b) If there were 10^4 at the end of 3 hours, how many were there at the beginning? Give your answer to the nearest tens (3mks)

(iv) Solve the differential equation:

$$\frac{dy}{dx} = \frac{y}{x} + x\sin\frac{y}{x}$$ (5mks)

QUESTION TWO

One of the basic equations in electric circuits is $L\frac{di}{dt} + R_i = E(t)$…… (i)

Where L is inductance, R (ohms) is the resistance, i (amperes) the current and E (volts) the electromotive force or emf.

(a) Solve equation (i), given that when t=0, $E(t) = E_0$ and $i = i_0$

(b) If $L = 3, R = 15$ and $E(t) = 110\sin 120\pi t$ and $i = 0$ when $t = 0$ show that a particular solution for equation (i) is given as:

$$i = 22/3\left[\frac{\sin 120\pi t - 24\pi\cos 120\pi t + 24\pi e^{-5t}}{1 + 576\pi^2}\right]$$ (20mks)

QUESTION THREE

(a) Solve the differential equation:

$xy\,dy/dx = x^2 + y\ + 2xy,$ given that when $x = 1, y = 0$ (8mks)

(b) A particle moves along a straight line such that its displacement x from a fixed point P is given by: $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = \cos 2t$

Using the differential operator D, find its displacement x at any time t, given that at $t = 0, \frac{dx}{dt} = 0$ (12mks)

QUESTION FOUR

(a) Solve the following simultaneous differential equations:

$$dx/dt + dy/dt + 2x + y = 0$$

$$dy/dt + 5x + 3y = 0$$ (10mks)

(b) Show that the solution of differential equation $(2x - 5y + 3)dx - (2x + 4y - 6)dy = 0$ can be expressed as $(4y - x - 3)(y + 2x - 3)^2 = B$ (10mks)

QUESTION FIVE

The differential equation satisfied by a beam of uniformly loaded (wkg/m) with one end fixed and the second end subjected to tensile force p is given by:

$$EI\ d^2y/dx^2 = P_Y - 1/2\, wx^2$$

Show that the elastic curve for the beam with conditions y = 0, $\frac{dy}{dx} = 0,$ at x = 0

$y = w/Pn^2\,(1 - Cosh\,nx) + \frac{wx^2}{2P}$, where $n^2 = \frac{P}{EI}$

(use method of undetermined coefficient.) (20mks)