KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 322**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATION II**

STREAM: **SESSION VI**

DAY: **THURSDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **12/08/2010**

INSTRUCTIONS:

Answer Question ONE and any other TWO Questions

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

a). Obtain two linearly independent solution of the differential equation

$4xy'' + 2y' + y = 0$ valid near the origin for x > 0 using power series method

(8 marks)

b). Write the following differential equations as a first order system:

i. $y'' + py' + qy = f(x)$

ii. $y''' + py'' + qy' + ry = f(x)$ (4 marks)

c). Fine two linear independent solutions to the system; $X' = AX$ where $A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$

(6 marks)

d). Use elementary elimination calculus to solve the following system of first order differential equation: $\begin{aligned} x' &= 3x - 4y \\ y' &= 4x - 7y \end{aligned}$ (6 marks)

e). Solve the following uncoupled system find the corresponding diagonal system of the equations below and solve. (6 marks)

$$\begin{aligned} x' &= -x - 3y \\ y' &= 2y \end{aligned}$$

QUESTION TWO (20 MARKS)

a). Solve the system; $x^1 = Ax + B$ where $A = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & e^{2t} \\ t & e^{2t} \end{pmatrix}$ (10 marks)

b). Find the general solution of the system

$$\begin{aligned} & D^2 y + (D-1)v = 0 \\ & (2D-1)y + (D-1)w = 0 \\ & (D+3)y + (D-4)v + 3w = 0 \end{aligned}$$

(10 marks)

QUESTION THREE (20 MARKS)

a). a). Check that $(x(t),\ y(t)) = (e^{-4t} - 3e^{-2t},\quad -3e^{-4t} + 3e^{-2t})$ is a solution of the system of differential equations below and test for their linear independence. (8 marks)

$$x' = -x + y$$
$$y' = -3x - 5y$$

b). Reduce the following differential equation into a first order system:

$y^{(iv)} - y = 0$ (4 marks)

c). Identify and classify all singular points of:

i. $4xy^{11} + 2y^{1} + y = 0$

ii. $9x(1-x)y^{11} - 12y^{1} + 4y = 0$

iii. $(x - x^2)y^{11} + (1 - x)y^{1} - y = 0$ (8 marks)

QUESTION FOUR (20 MARKS)

a). Show that $f_1(x) = \cos x$ and $f_2(x) = \sin x$ are linearly independent solutions of the differential equation $y'' + y = 0$ (6 marks)

c). Solve the initial value problem $(1 + y^2)dx = (1 + x^2)dy = 0$ With the initial conditions that when $x = 0, y = 1$. (6 marks)

d). By using suitable transformation to reduce the equation to a separable equation, solve

$(x + y)dx + (3x + 3y - 4)dy = 0$ (8 marks)

QUESTION FIVE (20 MARKS)

a). Use elementary elimination calculus to solve the following system of first order differential equation:

$$y' = y$$
$$y' = -2x + 3y$$

(6 marks)

b). By use of a suitable integrating factor solve the differential equation

$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ (8 marks)

c). Show that $e^{ax}, xe^{ax}, x^2e^{ax}$ are solutions of the differential equation

$\frac{d^3y}{dx^3} - 3a\frac{d^2y}{dx^2} + 3a^2\frac{dy}{dx} - a^3y = 0$ hence find the general solution of this equation.

(6marks)