KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 410

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS

STREAM: SESSION VII

DAY: MONDAY

TIME: 2.00 – 4.00 P.M.

DATE: 29/11/2010

INSTRUCTIONS:

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

a) Calculate the integral surface of the quasi - linear partial differential equation

$x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$ which contains the straight line

$x+y=0,\ z=1$ **(6 marks)**

b) State whether the differential equation is linear, its order and degree

$r+3s+t=0$ where $r=\dfrac{\partial^2 z}{\partial x^2}$, $s=\dfrac{\partial^2 z}{\partial x\partial y}$ and $t=\dfrac{\partial^2 z}{\partial y^2}$ **(3 marks)**

c) Show that the partial differential equation $xp-yq=x,$ and $x^2p+q=xz$ are compatible and find their solution. **(6 marks)**

d) Obtain the first order partial differential equation from the relation $z=ax^6y^3+bx^4y^2+cx^2y+d$. **(7 marks)**

e) Find a complete and singular integral of $2xz-px^2-2qxy+pq=0$

(8 marks)

QUESTION TWO (20 MARKS)

a) Calculate the integral surface of the quasi - linear partial differential equation

$$x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$$ **(8 marks)**

b) Classify each of the following partial differential equations as either parabolic, elliptic or hyperbolic

i). $u_{xx}-x^2yu_{yy}=0,\ \ (y>0)$,

ii). $4u_{xx}+12u_{xy}+9u_{yy}-2u_x+u=0$

iii). $2u_{xx}+2u_{yy}-15u_{zz}+8u_{xy}-12u_{xy}-12u_{yz}=0$ **(6 marks)**

c) Verify that the following equation is integrable and determine their primitives: $zydx-zxdy-y^2dz=0$ **(6 marks)**

QUESTION THREE (20 MARKS)

a) Find the fundamental solution of the Laplace's equation
$\Delta u = \frac{d^2u}{dr^2} + \frac{(n-1)}{r}\frac{du}{dr} = 0$ **(8 marks)**

b) Solve $\frac{dx}{6(y-z)} = \frac{2dy}{3(z-x)} = \frac{3dz}{2(x-y)}$ **(6 marks)**

c) Solve the Lagrange's equation: $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$
(6 marks)

QUESTION FOUR (20 MARKS)

a) Use Charpits methods to find the complete integral of the partial differential equations $p^2x + q^2y = z$ **(6 marks)**

b) Determine the constant k such that the equation
$\left(\frac{1}{x^2} + \frac{1}{y^2}\right)dx + \left(\frac{kx+1}{y^3}\right)dy = 0$ is exact. **(4 marks)**

c) Obtain the partial differential equation of the following equations where **a, b** and **c** are constants and ϕ is arbitrary function to be eliminated.
   (i) $z = ax + by + cxy$
   (ii) $y = f(x - at) + \phi(x + at)$ **(10 marks)**

QUESTION FIVE (20 MARKS)

a) Calculate the equation of the tangent plane at the point (2, 1, -2) to the surface
$x^2 + 2y^2 + 2z^2 = 14$ **(7 marks)**

b) Find the surface orthogonal to the family of surfaces $z = cxy(x^2 + y^2)$ which passes through the curve $x^2 - y^2 = a^2, z = 0$. **(8 marks)**

c) The acceleration of a particle moving in straight line is the negative of its velocity. If it starts from the origin with a velocity of 1, find its position at the end of two units of time. **(5 marks)**