KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 410

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS 1

STREAM: SESSION VII & VIII

DAY: MONDAY

TIME: 2.00 – 4.00 P.M.

DATE: 10/08/2009

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

a. Find the equation of the tangent plane to the surface $z = 2x^2 + y^2$ at the point (1,1,3) (4 marks)

b. Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of its curves of intersection with the planes parallel to the plane z = 0. (5 marks)

c. Find the integral surfaces of the following differential equation:

$$\frac{dx}{x(3y-4z)} = \frac{dy}{y(4z-2x)} = \frac{dz}{z(2x-3y)}$$ (5 marks)

d. Determine whether the following differential equation is integrable and hence find the primitive:

$y\,dx - (x + z)\,dy + y\,dz = 0$ (5 marks)

e. Form a first order PDEs by eliminating the arbitrary constants:

(i) $z = f(x^2 - y^2)$ (3 marks)

(ii) $ax^2+by^2+z^2= 1$ (3 marks)

f. Show that the following equations are compatible and hence solve:
$(xp - yq) = x,\ x^2p + q = xz$ (5 marks)

QUESTION TWO (20 MARKS)

a. Find the general solution u(x,y) of $u_{xx} - u = 0$ which satisfies the auxiliary conditions $u(0, y) = f(y),\ u_x(0, y) = g(y)$ (5 marks)

b.

Find the equation of the tangent plane to:

$xyz=a^3$ at the point (x_0,y_0,z_0) (4 marks)

c. Find the orthogonal trajectories on the surface $y^2 = 2z$ which is cut by the system of planes x + z = c where c is a constant.

(5 marks)

d. Show that $u = f(x^2 - y^2)$ is a solution of $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$ (4 marks)

e. Define a compatible system of first order equation. (2 marks)

QUESTION THREE (20 MARKS)

a. Find the integral surfaces of the following differential equations:

(i) $\frac{dx}{cy-bz} = \frac{dy}{az-cx} = \frac{dz}{bx-ay}$ (5 marks)

(ii) $\frac{dx}{y^3x-2x^4} = \frac{dy}{2y^4-x^3y} = \frac{dz}{2z(x^3-y^3)}$ (5 marks)

(iii) $\frac{dx}{x(4y^2-z^2)} = \frac{-dy}{y(z^2+9x^2)} = \frac{dz}{z(9x^2+4y^2)}$ (5 marks)

b. Use separation of variables method to solve the following Pfaffian differential equations:

$y(1-x)dx + x^2(1-y)dy = 0$ (5 marks)

QUESTION FOUR (20 MARKS)

a. Verify that the following differential equations are integrable, homogeneous and hence find the primitives:

(i) $yz(y+z)\,dx + xz(x+z)\,dy + xy(x+y)\,dz = 0$ (6 marks)

(ii) $(2x + y)\,dy - (x - 2y)\,dx = 0$ (6 marks)

b. Determine whether the following differential equations are integrable and hence find their primitives:

(i) $y\,dx + x\,dy + 2z\,dz = 0$ (4 marks)

(ii) $2xz\,dx + z\,dy - dz = 0$ (4 marks)

QUESTION FIVE (20 MARKS)

a. Form PDEs by eliminating the arbitrary constants in following equations:

(i) $x^2 + y^2 = (z - c)^2 \tan^2 p$ (4 marks)

(ii) $z = x + y + f(xy)$ (4 marks)

b. Use Cauchy's method to find the solutions of the following PDE satisfying the given conditions:

$pq = z$, $x = 0$, $y^2 = z$ (7 marks)

c. Use Charpit's method to solve the following first order PDE:

$(p^2 + q^2)\,y = qz$ (5 marks)