KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 410**

COURSE TITLE: **PARTAIL DIFFERENTIAL EQUATIONS**

STREAM: **Y4S1**

DAY: **FRIDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **06/08/2010**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** questions

PLEASE TURNOVER

QUESTION ONE: 30 MARKS

a) Given that $z = x^3 y + y^2 x$, Find dz. (3 Marks)

b) Show that the equation $x = 2u, y = v$ and $z = u^2 + v^2$ represent a surface and find its equation in constraint forms. (4 Marks)

c) Find the equations of the tangent plane and the normal line to the surface $z = \sin x + \cos y$ at a point $(\pi, \frac{\pi}{2}, 1)$. (6 Marks)

d) Solve the equation $\frac{dx}{6(y-z)} = \frac{2dy}{3(z-x)} = \frac{3dz}{2(x-y)}$ (6 Marks)

e) Determine b so that the differential equation $(3x - 5y + 7)dx + (bx + 6y + 10)dy$ will be exact and solve it. (5 Marks)

f) Form a quasi-linear partial differential equation of order one whose general solution is given by
$Q(x^2 - y^2, y^2 + 2z^2) = 0$. (4 Marks)

g) Solve the non-linear partial differential equation $p^2 + q^2 = 1$. (2 Marks)

QUESTION TWO: 20 MARKS

a) The surfaces $3x^{2y} + y^2 z + z^2 = 0$ and $2xz - x^2 y = 3$ intersect in a space curve. Find the equation of the tangent line to this curve at the point $(-1, 2)$. (7 Marks)

b) Find the integral surface of the equation $x(3y - 4z)p + y(4z - 2x)q = z(2x - 3y)$ which passes through the line $y = 2x, z = 1$. (13 Marks)

QUESTION THREE: 20 MARKS

a) Find the integrating factor of the differential equation: $ydx + (y^2 - x)dy = 0$ and hence obtain its general solution. (7 Marks)

b) Solve the following homogeneous equation :
$(2yz + 3xy + 4x^2)dx + (xz + x^2)dy + xydz = 0$ (10 Marks)

c) Use separation of variables method to solve the differential equation, $e^{x^3 - y^2} + \frac{y}{x^2}\frac{dy}{dx} = 0$.
(3 Marks)

QUESTION FOUR: 20 MARKS

a) Given that $p_o(x_0, y_0, z_0)$ is a point on a curved surface whose equation is $f(x, y, z) = 0$. Derive the equation of the tangent plane and the normal line at the point $p_o(x_0, y_0, z_0)$.

(10 Marks)

b) Find the equation of the tangent plane and the normal line to the surface $x = u, y = \frac{v}{2}, z = uv$ at the point $(-2, 1, 2)$ (8 Marks)

c) Determine the constant K such that the equation $(\frac{1}{x^2} + \frac{1}{y^2})dx + (\frac{kx+1}{y^3}) = 0$ is exact.(2 Marks)

QUESTION FIVE: 20 MARKS

a) Show that the partial differential equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible.

(10 Marks)

b) Obtain a first order partial differential equation from the relation
$z = ax^6y^3 + bx^4y^2 + cx^2y + d$. (5 Marks)

c) Find the general solution of the equation $y^2zp + z^2xq = -xy^2$ (5 Marks)