

KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF ECONOMICS &
MATHEMATICS

COURSE CODE: MATH 410

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS

STREAM: Y4S1

DAY: MONDAY

TIME: 11.00 – 1.00 P.M.

DATE: 08/12/2008

INSTRUCTIONS TO CANDIDATES:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

- (a). Find the equation of the tangent plane to the surface $3xyz - z^3 = a^3$ at $(0, a, -a)$. (4 marks)
- (b). Verify that the following equation is integrable and determine their primitives: $zydx - zxdy - y^2dz = 0$ (4 marks)
- (c). Find the integral curves to the equations: $\frac{dx}{6(y-z)} = \frac{2dy}{3(z-x)} = \frac{3dz}{2(x-y)}$. (5 marks)
- (d). Form a first order PDE from the equation: $x^2 + y^2 + (z-c)^2 = a^2$ (5 marks)
- (e). Use Jacobi's method to solve the first order PDE: $p+q = pq$ (5 marks)
- (f). State what is a first order PDE. (2 marks)
- (g). Find the orthogonal trajectories on the surface $y^2 = 2z$ which is cut by the system of planes $x + z = c$ where c is a constant. (5 marks)

QUESTION TWO (20 MARKS)

- (a). What is PDE? (2 marks)
- (b). Find the orthogonal trajectories on the surface $(x+y)z = 1$ which is cut by the planes $x-y + z = k$ where k is a constant. (6 marks)
- (c). Find the integral curves of the following simultaneous differential equations:

(i) $\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$ (7 marks)

(ii) $\frac{dx}{cy-bz} = \frac{dy}{az-cx} = \frac{dz}{bx-ay}$ (5 marks)

QUESTION THREE (20 MARKS)

(a). Use Cauchy's method to find the solutions of the P.D.E
 $xp-yq+z = x, z = x^2$ when $y = x$. (8 marks)

(b). Find a complete integral of the equation $p + q = pqz$ (5 marks)

(c). Use Charpit's method to find the complete integral of the equation

$$p^2 z^2 = 1 - q^2. \quad (7 \text{ marks})$$

QUESTION FOUR (20 MARKS)

(a). Explain what you understand by the term "compatible equations" in PDE. (3 marks)

Hence show that the equations $xp - yq = x, x^2 p + q = xz$ are compatible and find their solution. (6 marks)

(b). Solve the Lagrange's equation: $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$ (6 marks)

(c). Use separation of equations method to solve the following equation:
 $p - x^2 = q + y^2$ (5 marks)

QUESTION FIVE (20 MARKS)

- (a). Verify that the following equation is integrable, homogeneous and hence find the primitive: $(2yz + 3xy + 4x^2)dx + (xz + x^2)dy + xydz = 0$ (8 marks)
- (b). Proof that the following differential equation is integrable and hence find the Primitive $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ (8 marks)
- (c). Find the direction ratios to the curves: $ax^2 + by^2 + cz^2 = 1, x+y+z = 1$ (4 marks)