KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 410

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS

STREAM: SESSION VII

DAY: WEDNESDAY

TIME: 2.00 – 4.00 P.M.

DATE: 11/08/2010

INSTRUCTIONS:

Answer Question ONE and any other TWO of the remaining.

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

a). State what is a first order PDE. (2 marks)

b). Verify that the following equation is integrable and determine their primitives: $zydx - zxdy - y^2dz = 0$ (4 marks)

c). The acceleration of a particle moving in straight line is the negative of its velocity. If it starts from the origin with a velocity of 1, find its position at the end of two units of time. (5 marks)

d). Calculate the equation of the tangent plane at the point (2, 1, -2) to the surface

$$x^2 + 2y^2 + 2z^2 = 14$$ (5 marks)

e). Use Charpits methods to find the complete integral of the partial differential equations

$$p^2x + q^2y = z$$ (8 marks)

Determine the value of b so that the differential equation

$$(3x - 5y + 7)dx + (bx + 6y + 9)dy = 0$$

will be exact and solve it. (6 marks)

QUESTION TWO (20 MARKS)

a). Find the integrating factor of the following differential equation and solve it.

$$2x^2ydx + (x^3 + 2xy)\,dy = 0$$ (6 marks)

b). Find the equation of the tangent plane to the surface

$x^2 + y^2 + z^2 = 18$ at the point (3, 3, 0) (6 marks)

c). Calculate the integral surface of the quasi - linear partial differential equation

$x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$ which contains the straight line $x + y = 0,\ z = 1$

(8 marks)

QUESTION THREE (20 MKS)

a). Use Charpits methods to find the complete integral of the partial differential equations

$$px + qy = pq$$ (8 marks)

b). Obtain the partial differential equation of the following equations where **a, b** and **c** are constants and ϕ is arbitrary function to be eliminated.

(i) $z = ax + by + cxy$ (4 marks)

(ii) $y = f(x-at) + \phi(x+at)$ (5 marks)

c). Solve $(x+y)p + (z+x)q = x+y$ (5 marks)

QUESTION FOUR (20 MKS)

a). Calculate the integral surface of the quasi - linear partial differential equation

$$x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$$

which contains the straight line $x+y=0,\ z=1$ (8 marks)

b). Classify the differential equations as either parabolic, elliptic or hyperbolic

i). $u_{xx} + yu_{yy} = 0$ (3 marks)

ii). $u_{xx} - 3u_{xy} + 2u_{yy} = 0$ (3 marks)

c). Find a complete and singular integral of $2xz - px^2 - 2qxy + pq = 0$ (6 marks)

QUESTION FIVE (20 MARKS)

a). Verify that the following equation is integrable, homogeneous and hence find the primitive: $(2yz+3xy+4x^2)dx + (xz+x^2)dy + xydz = 0$ (8 marks)

b). Solve by separation of variables the differential equation $(x^2+1)(y^2-1)dx + xydy = 0$ (6 marks)

c). State whether the differential equation is linear, its order and degree $r+3s+t=0$ where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$ and $t = \frac{\partial^2 z}{\partial y^2}$ (6 marks)