KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 410**

COURSE TITLE: **PARTIAL DIFFERENTIAL EQUATION I**

STREAM: **Y4S1**

DAY: **TUESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **24/03/2009**

INSTRUCTIONS:

ANSWER **QUESTION ONE AND ANY OTHER TWO QUESTIONS**

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

a. What is P.D.E? **(1 mark)**

b. Explain what you understand by the term "compatible equations" in PDE. **(2 marks)**

c. Form a first order PDE from the equation $ax^2 + by^2 + z^2 = 1$ **(4 marks)**

d. Use Charpit's method to find the complete integral of the equation:

$p^2 z^2 = 1 - q^2.$ **(6 marks)**

e. Use Jacobi's method to find the complete solution of the differential equation

$zpx + y^2 q^2 - z^2 = 0.$ **(6 marks)**

f. Find the equation of the tangent plane to the surface $\frac{x^2}{16} + \frac{y^2}{9} = \frac{z^2}{8}$ at the point (4,3,4). **(6 marks)**

g. Find the complete integral of the PDE: zpq = p + q **(5 marks)**

QUESTION TWO (20 MARKS)

a. Define the following terms as used in PDE:

(i) Complete integral **(2 marks)**

(ii) General integral **(2 marks)**

b. Hence use Cauchy's method to find the complete integral of the equation

$(y^2 - z^2)p - xyq = xz$ containing x = y = z, x > 0. **(9 marks)**

c. Use charpit's method to find the complete integrals of the following differential equations:

$px^5 - 4q^3x^2 + 6x^2z - 2 = 0$ **(7 marks)**

QUESTION THREE (20 MARKS)

a. Find the integral curves of the following simultaneous differential equation:

(i) $\dfrac{dx}{2xz} = \dfrac{dy}{-2yz} = \dfrac{dz}{x^2 - y^2}$ **(5 marks)**

(ii) $\dfrac{dx}{x(4y^2 - 4z^2)} = \dfrac{-dy}{y(z^2 + 9x^2)} = \dfrac{dz}{z(9x^2 + y^2)}$ **(5 marks)**

b. Find the equations for the tangent plane to the surface

$x^2yz + 3y^2 - 2xz^2 = 8z$

at the point (1,2,-1). **(4 marks)**

c. Test for integrability. Hence find the primitive of $(x - x^2y)dy + ydx = 0$.

(6 marks)

QUESTION FOUR (20 MARKS)

a. Show that the following pair of surfaces are tangent at the point (3,3,0)

$x^2 + y^2 + z^2 = 18$

xy = 9

(7 marks)

b. Test whether the differential equation $p^2z^2y^2dx + q^2x^2z^2dy + r^2x^2y^2dz = 0$ is integrable. Hence solve using separation of variables method where p, q and r are constants. **(6 marks)**

c. Find the orthogonal trajectories of the surfaces: $3x^2 + 4y^2 + 8z^2 - 36 = 0$ and $x^2 + 2y^2 - 4z^2 - 6 = 0.$ (7 marks)

QUESTION FIVE (20 MARKS)

a. Verify that the following differential equations are homogeneous and integrable and hence find the primitive :

$yzdx + (x^2y - 2x)dy + (x^2z - xy)dz = 0$ **(10 marks)**

b. Verify whether the following equation is integrable and hence solve:

$zy^2dx + zx^2dy - x^2y^2dz = 0$ (6 marks)

c. Solve the pfaffian differential equation $e^{x2-y2} + \dfrac{y}{x^2}\dfrac{dy}{dx} = 0$ using separation of variables.

(4 marks)