KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 410**

COURSE TITLE: **PARTIAL DIFFERENTIAL EQUATIONS**

STREAM: **Y4S1**

DAY: **MONDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **10/08/2009**

INSTRUCTIONS TO CANDIDATES:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

a. Find the integral curves of the following differential equation:

$$\frac{dx}{x(4y^2 - z^2)} = \frac{-dy}{y(z^2 + 9x^2)} = \frac{dz}{z(9x^2 + 4y^2)}$$ (5 marks)

b. Find the general solution u(x,y) of $u_{xxx} + u_x = 0$ (3 marks)

c. Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of its curve s of intersection with the planes parallel to the plane z = 0.

(5 marks)

d. Solve the following homogeneous differential equation:

$$(y^3+2xyz)dx+(4xy^2+2x^2z)dy + x^2y = 0$$ (5 marks)

e. Determine whether the following differential equation is integrable and hence find the primitive:2xz dx + z dy – dz = 0

(6 marks)

f. Form PDEs by eliminating the arbitrary constants in following equations:

(i) $z = y^2+f(\frac{1}{x}+\ln y)$ (3 marks)

(ii) $x^2+y^2+(z-c)^2 = a^2$ (3 marks)

QUESTION TWO (20 MARKS)

a. Verify that v = sinwct sinwx is a solution of the equation: $\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2}$

(5 marks)

b. Show that $u = f(x^2 - y^2)$ is a solution of $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$ (5 marks)

c. Find the equation of the tangent plane to the surface $z = 2x^2 + y^2$ at the point (1,1,3)

(5 marks)

d. Find the orthogonal trajectories on the surface $(x+y)z = 1$ which is cut by the planes $x - y + z = k$ where k is a constant.

(5 marks)

QUESTION THREE (20 MARKS)

a. Form first order PDEs by eliminating the arbitrary constants in the following:

(i) $2z = (ax+y)^2 + b$ (5 marks)

(ii) $z = xy + f(x^2+y^2)$ (3 marks)

b. Use Charpit's method to solve the following first order PDEs:

(i) $2zx - px^2 - 2qxy + pq = 0$ (5 marks)

(ii) $(p^2 + q^2)y = qz$ (7 marks)

QUESTION FOUR (20 MARKS)

a. Solve the following linear PDEs:

(i) $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

(5 marks)

(ii) $(y^3 - 2x^3)xp + (2y^3 - x^3)yq = 9z(x^3 - y^3)$

(5 marks)

b. Solve the following special types of PDEs:

(i) $z^2(p^2 + q^2 + 1) = 1$

(6 marks)

(ii) $zpq = p + q$

(4 marks)

QUESTION FIVE (20 MARKS)

a. Use Jacob's method to solve the following PDEs:

(i) $p^2q^2 + x^2y^2 = x^2q^2(x^2+y^2)$

(5 marks)

(ii) $p + q = pq$ (5 marks)

b. Use Cauchy's method to find the solutions of the following PDEs satisfying the given conditions:

(i) $p + q - z = 0$, $z = 1 + \cos x$ when $y = 2x$ (5 marks)

c. Find the general integrals of the following linear PDE:
$z(xp - yq) = y^2 - x^2$ (5 marks)