KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 410

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS

STREAM: **SESSION VII**

DAY: **FRIDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **28/11/2008**

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** of the remaining.
2. All the working **MUST** be shown clearly

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) The acceleration of a particle moving in straight line is the negative of its velocity.

If it starts from the origin with a velocity of 1, find its position at the end of two units of time. **(5 mks)**

(b) Show that the surface $x^2 + 2yz + y^3 = 4$ is perpendicular to any member of the family the surface $x^2 + 1 = (2 - 4a)y^2 + az^2$ at a point of intersection (1, -1, 2) **(5 mks)**

(c) Solve the equation

$$(y + z)p + yq = x - y$$ **(5 mks)**

(d) Verify that the following Pfaffian differential equation is intergrable and solve

$$ydx - (x + z)dy + ydz = 0$$ **(5 mks)**

(e) Find the surface integrals of the following equation

$$\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{z}$$ **(5 mks)**

(f) Calculate the equation of the tangent plane at the point (2, 1, -2) to the surface

$$x^2 + 2y^2 + 2z^2 = 14$$ **(5 mks)**

QUESTION TWO (20 MARKS)

(a) Obtain the partial differential equation of the following equations where **a** and **b** are constants.

(i) $z = (x + a)(y + b)$ **(4 mks)**

(ii) $\phi = (ye^z, x^2e^z) = 0$ **(5 mks)**

(b) Verify whether the following equation is integrable, and hence of otherwise solve;-

$$zy^2dx + zx^2dy - x^2y^2dz = 0$$ **(6 mks)**

(b) Show that the following Pfaffian differential equation

$$2x(x^3 + y^3)dx + 3(x^2y^2 + y^4)dy = 0$$

is an exact differential. Hence, or otherwise find its primitive. **(5 mks)**

QUESTION THREE (20 MKS)

(a) Calculate the integral surface of the quasi - linear partial differential equation

$$x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$$

which contains the straight line $x+y=0,\ z=1$ **(6 mks)**

(b) Verify whether the partial differential equation

$$xp - yq = 0$$

$$z(xp+yq) - 2xy = 0, \text{ are compatible.}$$

Hence or otherwise find their solution **(7 mks)**

(c) Use Charpits methods to find the complete integral of the partial differential equations

$$p^2x + q^2y = z$$ **(7 mks)**

QUESTION FOUR (20 MKS)

(a) Determine orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersection with the family of planes parallel to $z=0$ **(10 mks)**

(b) (i) Determine the value of b so that the differential equation

$$(3x-5y+7)dx + (bx+6y+9)dy = 0$$

will be exact and solve it. **(4 mks)**

(ii) Find the integrating factor of the following differential equation and solve it.

$$2x^2y\,dx + (x^3+2xy)\,dy = 0$$ **(6 mks)**

QUESTION FIVE (20 MARKS)

(a) Find the integral curves of the following simultaneous differential equations;

(i) $\dfrac{dx}{2xz} = \dfrac{dy}{2yz} = \dfrac{dz}{z^2 - x^2 - y^2}$ **(5 mks)**

(ii) $\dfrac{dx}{x+y} = \dfrac{dy}{x+y} = \dfrac{dz}{-(x+y+2z)}$ **(5 mks)**

(c) (i) Find the equation of the tangent plane to the surface

$x^2 + y^2 + z^2 = 18$ at the point (3, 3, 0) **(5 mks)**

(ii) Show that the surfaces defined by $x^2 + 4y^2 - 4z^2 - 4 = 0$ and

$x^2 + y^2 - z^2 - 6x - 6y + 2z + 10 = 0$ are tangent at point (2, 1, 1) **(5 mks)**

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