

KABARAK



UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION

SCIENCE

COURSE CODE: MATH 410

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS

STREAM: SESSION VII & VIII

DAY: THURSDAY

TIME: 2.00 – 4.00 P.M.

DATE: 09/04/2009

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

PLEASE TURN OVER

QUESTION ONE(30 MARKS)

a. Calculate the integral curves of the following equations:

$$(i) \quad \frac{dx}{6y-6z} = \frac{2dy}{3z-3x} = \frac{3dz}{2x-2y} \quad (5 \text{ marks})$$

$$(ii) \quad \frac{dx}{-xz^3+xy^3} = \frac{dy}{yz^3-yx^3} = \frac{dz}{-zy^3+x^3z} \quad (5 \text{ marks})$$

b. Find the first order PDE whose complete integral is: $(x-a)^2 + (y-b)^2 = z^2$

where a,b and α are constants. (5 marks)

c. Find the orthogonal trajectories on the surface $2y - x^2 = 0$ orthogonal to its intersection with the system $xy = z + c$.

(5 marks)

d. Solve the following homogeneous Pfaffian differential equation:

$$(2x - y) dy - (x - 2y) dx = 0 \quad (6 \text{ marks})$$

f. Show that the function $u = f(x^2 + y^2)$ where f is an arbitrary function is a solution of

$$yu_x - xu_y = 0 \quad (4 \text{ marks})$$

QUESTION TWO (20 MARKS)

a. Verify whether the following equations are integrable and hence solve:

$$(i) \quad (y dx + xdy)(a-z)+xydz=0 \quad (5 \text{ marks})$$

$$(ii) \quad (y^2 - z^2)dx + (x^2 - z^2)dy + (x + y)(x + y + 2z)dz = 0 \quad (5 \text{ marks})$$

b. Verify that the following differential equations are homogeneous and hence solve:

(i) $yz(y+z)dx+xz(x+z)dy+xy(x+y)dz=0$ (5 marks)

(ii) $(2yz + 3xy + 4x^2)dx + (xz + x^2)dy + xydz = 0$ (5 marks)

QUESTION THREE (20 MARKS)

a. Obtain the first order PDEs of the following equations where a and b are constants:

(i) $z = (x+a)(y+b)$ (5 marks)

(ii) $ax^2 + by^2 + z^2 = 1$ (5 marks)

(iii) $2z = (ax + y)^2 + b$ (5 marks)

b. Use charpit's method to find the complete integrals of the following differential

equation: $(p^2 + q^2)y = qz$ (5 marks)

QUESTION FOUR (20 MARKS)

a. Use Cauchy's method to find the solutions of the following PDEs satisfying the prescribed conditions: $p - yq + z = x$, $z =$ when $y = x$ (10 marks)

b. Find the general solution of the equation $(y+z)p + yq = x - y$ whose integral surface contains the curve $y = 1$, $z = 1 + x$, $-\infty < x < \infty$. (10 marks)

QUESTION FIVE (20 MARKS)

a. Use Jacobi's method to find the complete integrals of the following equation:

$$z^2 + p^2 + q^2 = 1 \quad (5 \text{ marks})$$

b. Find a complete integral of the PDE: $z = p^2 - q^2$ (5 marks)

c. Find a complete integral of the Clairaut's equation: $(p+q)(z-xp-yq)=1$ (5 marks)

d. Find a complete integral of the following PDE: $(p^2 + q^2)z = p^2 + q^2$ (5 marks)