

2008/2009 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

## COURSE CODE: MATH 329

COURSE TITLE: QUALITY CONTROL AND ACCEPTANCE

STREAM:
Y3S2
DAY:
THURSDAY
11.00-1.00 P.M.

DATE:
11/12/2008

INSTRUCTIONS:

1. Question ONE is compulsory.
2. Attempt question ONE and any other TWO

## PLEASE TURN OVER

## QUESTION ONE-COMPULSORY (30 marks)

(a) Explain the meaning of the following terms as used in statistical quality control
i. Assignable cause
ii. Average Run Length
iii. Lot Tolerance Percent Defective
iv. Average Outgoing Quality
(8 marks)
(b) Derive the expression for the $3 \sigma$ control limits of a random sample $x_{1}, \ldots, x_{n}$ obtained from a random variable X , when $\delta\left(x_{1}, \ldots, x_{m}\right)$ is a function of the sample and the random variable X is not from a normal distribution.
(3 marks)
(c) The two common charts used in manufacturing processes are $\bar{x}-$ chart and R-charts. $\bar{x}-$ charts detect undesirable variation in process average ( $\mu$ ), while R-charts detect undesirable variation in process variability $\left(\sigma^{2}\right)$
i. When is the process considered to be under statistical control?
(1 mark)
ii. Set up the $3 \sigma$ control limits for the $\bar{x}$ - chart when both the mean $(\mu)$ and variance $\left(\sigma^{2}\right)$ of the population are unknown. Show clearly how to estimate the unknown population parameters. Assume normality holds.
(8 marks)
(d) Control charts for $\bar{x}$ and R are maintained on a certain dimension of a manufactured part, measured in meters. The subgroup size is five. The value of $\bar{x}$ and R are computed for each subgroup and recorded in a table as shown below.

| Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Average $\left(\bar{x}_{i}\right)$ | 143.0 | 139.8 | 139.2 | 140.0 | 139.2 | 141.4 | 140.6 | 140.4 | 141.8 | 138.0 |
| Range $\mathbf{R}_{\mathbf{i}}$ | 8 | 6 | 8 | 8 | 13 | 9 | 8 | 13 | 8 | 6 |

Estimate the mean $(\mu)$ and variance $\left(\sigma^{2}\right)$ of the process assuming that it is under statistical control. Calculate the $3 \sigma$ control limits for the $\bar{x}$ and R-charts. Is the process under statistical control? $\quad\left[\mathrm{d}_{2}=2.326 \mathrm{D}_{3}=0 \mathrm{D}_{4}=2.11\right]$
(6 marks)
(e) Define the following terms as used in acceptance sampling
i. Acceptance Quality Level [AQL]
ii. Acceptance Outgoing Quality [AOQ]
(f) The statistical quality control (SQC) department is a necessary component of any production process. State two major roles of this department in a production process.

## QUESTION TWO (20 Marks)

(a) When measurable characteristics are many it is uneconomical to maintain the $\bar{x}$ charts and Rcharts. Control charts for attributes are preferred which include the p-chart. The process is
considered to be under statistical control if all the samples taken from the production process have the same proportion $\mathbf{P}$ as the population. Let N and n be the lot size and sample size respectively.
i. Set up the action and warning limits for p -chart for large sample size, taking $\alpha=0.2 \%$ and $\alpha=5 \%$ for the action and warning limits respectively.
(5 marks)
ii. Obtain the action and warning limits for a p-chart of the tube process where 100 units are inspected each day. The data for each day for tube type D is recorded in table below.
(5 marks)

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Fraction <br> defective | 0.22 | 0.33 | 0.18 | 0.24 | 0.18 | 0.27 | 0.31 | 0.24 | 0.29 | 0.31 |

(b) Determine the control limits for the number of defects per unit (c-chart) for large n and moderately small p at $\alpha$ level of significance?
(4 marks)
(c) A record was made on the number of defects found in a laboratory equipment each hour for 12 hours as shown in the following table

| Hour | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of defects | 8 | 1 | 7 | 2 | 5 | 7 | 6 | 6 | 9 | 4 | 3 | 12 |

Determine the action and warning limits for the c-chart taking $\alpha=0.2 \%$ and $\alpha=5 \%$ for the action and warning limits respectively. Use normal approximation to the Poisson distribution. Is the process is under statistical control.
(6 marks)

## QUESTION THREE (20 marks)

(a) Describe the single sampling plan procedure.
(3 marks)
(b) A lot has the incoming quality $\theta$ and size N . A sample of size n is selected and we accept the lot if the number of defective items is less or equal to c items. Now derive the expression of operation characteristic. Further, obtain the expression of producer's risk and consumer's risk if they claim lot of quality $\theta_{1}$ and $\theta_{2}$ respectively.
(6 marks)
(c) Find the values of operating characteristic $p(\theta)$ when $\theta_{2}=L T P D$ and $\theta_{1}=A O L$ if the risk assumed by the producer and consumer are $\alpha=0.05$ and $\mathrm{B}=0.10$ respectively.
(6 marks)
(d) Compute for the approximate values of the producer's and consumer's risks in a single sampling inspection plan with $\left[\mathrm{n}=6, \theta_{1}=0.2, \theta_{2}=0.2, \mathrm{~N}=\infty\right.$ ]

## QUESTION FOUR (20 Marks)

a) Name any two types of acceptance inspection procedures and describe them fully.
b) Explain the following terms as used in acceptance sampling
i. Producer's risk
ii. Operating Characteristics (OC) function
iii. Sampling by inspection
(6 marks)
c) A random sample of size n is selected from a normal population of mean $(\mu)$ and variance $\left(\sigma^{2}\right)$. Suppose the process mean $(\mu)$ has change by $\theta$ but variance unchanged. Derive the expression of the probability that the process is under control denoted by $p(\theta)$ for the $\bar{x}$ chart, $\alpha=0.2 \%$
(5 marks)
d) Sketch a graph of $p(\theta)$ against $\theta$ for part (c) above.
e) A process is normally distributed with mean 100 and standard deviation of 5. Find probability of detecting a change of process mean to 108 , using the x -chart. ( $\mathrm{n}=5, \alpha=0.2 \%$

## QUESTION FIVE (20 Marks)

a) The following is a record of number of defects per unit in textile product $\mathbf{1 7 , 8 , 1 8 , 1 3 , 1 0 , 1 5 , 1 5 , 1 4 , 1 3 , 9 , 1 2 , 7 , 1 1 , 8 , 1 4 , 8 , 1 2 , 1 1 , 1 2 , 1 3}$ Construct a $3 \sigma$ c-chart and comment on the state of the process.
(4 marks)
b) From a lot consisting of 2000 items a sample of size 2225 is taken. If it contains 14 or less defective then the lot is accepted, otherwise rejected.
i. Plot the OC curve.
ii. Plot ASN curve.
iii. Hence obtain AOQL (6 marks)
c) Draw the lines of rejection and acceptance. Label the regions on the graph.
d) Determine a sequential sampling plan with the following parameters

$$
\begin{equation*}
\theta_{0}=0.04, \alpha=0.02, \theta_{1}=0.2, \beta=0.02 \tag{5marks}
\end{equation*}
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