KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 221

COURSE TITLE: REAL ANALYSIS I

STREAM: Y2S2

DAY: FRIDAY

TIME: 9.00 - 11.00 A.M

DATE: 17/12/2007

INSTRUCTIONS

1. Answer Question **ONE** and any other **TWO** Questions
2. Show **ALL** your workings.

PLEASE TURN OVER

QUESTION ONE (30 MARKS) COMPULSORY

(a) Prove the following properties of an ordered field.

(i) $a + c = b + c$ implies $a = b$ **(2 mks)**

(ii) $a + x = a$, $\forall$ a ,then $x = 0$ **(2 mks)**

(iii) $ax = 1$, $\forall$ a then $x = a^{-1}$ **(2 mks)**

(b) show that A and B are non-empty sets of real numbers with C= A+B where;
$C=\{x + y: \ x\in A, y\in B\}$show that; *sup*C =*sup*A +*sup*B **(4 mks)**

(c) What do you understand form the following terminologies?

(i) Extended real number system **(1 mk)**

(ii) Continum property **(1 mk)**

(iii) Supremum **(1 mk)**

(iv) Infimum **(1 mk)**

(d) Prove that $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ is an irrational number **(5 mks)**

(e) Show that every subset of N numbers which is not empty has a minimum. **(5 mks)**

(f) State and prove the sandwich theorem for sequences. Hence prove that:
$\sqrt{n+1} - \sqrt{n} \rightarrow 0$ as $n \rightarrow \infty$ **(6 mks)**

QUESTION TWO (20 MARKS)

(a) Define a metric space. **(4 mks)**

(b) Show that

(i) if $E \subseteq F$ then $E^{o} \subseteq F$ **(3 mks)**

(ii) E is open iff $E = E^{o}$ **(3 mks)**

(c) Suppose that $X_n \longrightarrow L$ as $n \rightarrow \infty$ show that the following two conditions holds.

(i) $X_n \geq a$, [n = 1,2……..]; then $L \geq a$. **(5 mks)**

(ii) $X_n \leq b$ [n = 1,2……...]; then $L \leq b$. **(5 mks)**

QUESTION THREE (20 MKS)

(a) Define a divergent sequence. **(2 mks)**

(b) Show that a sequence $\{(-1)^n\}$ is bounded by 1 but is divergent. **(2 mks)**

(c) State and prove Cauchy's criterion for limits of functions. **(10 mks)**

QUESTION FOUR (20 MARKS)

(a) Let $\{X_n\}$ and $\{Y_n\}$ be two sequences converging to L and h respectively as $n \to \infty$ then show that:

(i) $X_n + Y_n \to L + h$ **(5 mks)**

(ii) $X_n Y_n \to Lh$ **(5 mks)**

(b) Show the uniqueness of a limit in a sequence. **(5 mks)**

(c) Show that a set S of real numbers is bounded iff $\exists$ a real number k such that $|x| \leq k$ $\forall x \in s$. **(5 mks)**

QUESTION FIVE (20 MARKS)

(a) State and prove the intermediate value theorem for continuous functions. (I.V.T) **(7 mks)**

(b) Let $f : D \to R$ be uniformly continuous on the set D and suppose $\{X_n)$ is the Cauchy sequence, then $f\{X_n\}$ is also Cauchy sequence in R. **(6 mks)**

(c) Let F be defined on an open interval J, which contains X_0. If f is differentiable at X_0, then show that f is continuous at X_0. **(7 mks)**

QUESTION SIX (20 MARKS)

a) Define real analysis. **(3 mks)**
b) Prove Abel's theorem 1= 0 **(14 mks)**
c) Are you ready for Real analysis II ? **(3 mks)**