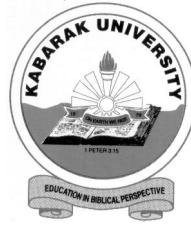


KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF SCIENCE IN
ECONOMICS & MATHEMATICS**

COURSE CODE: MATH 311

COURSE TITLE: REAL ANALYSIS II

STREAM: Y3S1

DAY: THURSDAY

TIME: 11.00 – 1.00 P.M

DATE: 11/12/2008

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Suppose that f is continuous on $[a, b]$ and differentiable on (a, b) then show that

(i) $f'(x) \geq 0 \quad \forall x \in (a, b)$ then f is increasing (5 mks)

(ii) $f'(x) \leq 0 \quad \forall x \in (a, b)$ then f is decreasing (5 mks)

(b) Let $\{x_0, x_1, \dots, x_n\}$ be a partition of $[a, b]$ such that $a = x_0 < x_1 < \dots < x_n = b$. Let $f(x)$ be increasing function defined on $[a, b]$. Show that

$$\sum_{k=1}^n \{f(x_k^+) - f(x_k^-)\} = f(b) - f(a) \quad (6 \text{ mks})$$

(c) Define the following terminologies

(i) a partition (3 mks)

(ii) Bounded variation (3 mks)

(d) If f is monotonic on $[a, b]$, then f is of bounded variation on $[a, b]$ (5 mks)

(e) Show that $S_n(x) = \frac{nx^2}{1+nx}$ converges uniformly to $S(x) = x$ on $A = [0, 1]$ (3 mks)

QUESTION TWO (20 MARKS)

(a) (i) Define the term Darboux Riemann integration. (3 mks)

(ii) Show that the function f defined on $[a, b]$ by:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$$

is not Riemann integrable on $[a, b]$ (6 mks)

(b) Prove that the function f is Riemann-steitejes integrable iff $\forall \epsilon > 0$

\exists a partition p on $[a, b]$ such that

$$U(f, p, \alpha) - L(f, p, \alpha) < \epsilon \quad (11 \text{ mks})$$

QUESTION THREE (20 MKS)

(a) If P^* is a refinement of P and $P \subseteq P^*$ where P is a partition of $[a, b]$, show that

$$L(f, P, \alpha) \leq L(f, P^*, \alpha) \quad (10 \text{ mks})$$

(b) Let $f \in B.V [a, b]$ and consider the function F defined on $[a, b]$ by

$$F(x) = \begin{cases} V_f(a, x) & \text{if } a < x \leq b \\ 0 & \text{if } a = x \end{cases}$$

show that F and $F - f$ are both increasing functions on $[a, b]$ (10 mks)

QUESTION FOUR (20 MKS)

(a) Let S be the interval $[a, b]$ if the sequence $\{f_n(x)\}$ of $R -$ integrable functions on S ,

then show uniformly to $f(x)$ on S , then show that $f(x)$ is $R -$ integrable. (10 mks)

(b) Let $f(x) = \begin{cases} x^2 \cos \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

then, show that f is B. V. on $[0, 1]$ if f is continuous on $[0, 1]$ and $f'(x)$ (5 mks)

(c) Define total variation. (5 mks)

QUESTION FIVE (20 MARKS)

(a) Show that if the series converges when $x = y$ then it converges for all x and x_0 is the

midpoint is the interval where $|x - x_0| = |y - y_0|$ holds. (12 mks)

(b) Let $f \in BV[a, b]$ and let $a < c < b$ show that $V_f[a, b] = V_f[a, c] + V_f[c, b]$ (8 mks)