KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/20010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 311

COURSE TITLE: REAL ANALYSIS II

STREAM: Y3S1

DAY: TUESDAY

TIME: 9.00 – 11.00 A.M.

DATE: 10/08/2010

INSTRUCTIONS:

- Attempt question **ONE** and any other **TWO** Questions

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

(a) Suppose that f is continuous on [a , b] and differentiable on (a, b) show that if

(i) $f^1(x) \geq 0 \quad \forall\ x \in (a,b)$ then f is increasing **(5 mks)**

(ii) $f^1(x) \leq 0 \quad \forall\ x \in (a,b)$ then f is decreasing **(5 mks)**

(b) Let $\{x_0, x_1, -----x_n\}$ be a partition of [a, b] such that $a = x_0 < x_1 < --- < x_n = b$.
Let $f(x)$ be increasing function defined on [a, b]. Show that

$$\sum_{k=1}^{n} \{f(x_k^+) - f(x_k^-)\} = f(b) - f(a)$$ **(6 mks)**

(c) Define the following terminologies

(i) a partition **(3 mks)**

(ii) Bounded variation **(3 mks)**

(d) Show that if f is monotonic on [a, b], then f is of bounded variation on [a, b] **(5 mks)**

(e) Show that $S_n(x) = \dfrac{nx^2}{1+nx}$ converses uniformly to $S(x) = x$ on $A = [0,1]$ **(3 mks)**

QUESTION TWO (20 MARKS)

(a) If P* is a refinement of P where P is a partition of [a, b], show that

$$\text{L}\ (f, p, \alpha) \leq L(f, p^*, \alpha)$$ **(10 mks)**

(b) Let $f \in$ B.V [a, b] and consider the function F defined on [a, b] by

$$\text{F(x)} = \begin{cases} V_f\,(a,x) & \text{if } a < x \leq b \\ 0 & \text{if } a = x \end{cases}$$

show that F and F – f are both increasing functions on [a, b] **(10 mks)**

QUESTION THREE (20 MKS)

(a) (i) Define the term Darboux Riemann intergration. **(3 mks)**

(ii) Show that the function f defined on [a, b] by:

$$f(x) = \begin{cases} 0 \text{ if x is irrational} \\ 1 \text{ if x is rational} \end{cases}$$

is not Riemann integrable on [a, b] **(6 mks)**

(b) Prove that the function f is Riemann-steitejes integrable iff $\forall \in > 0$

$\exists$ a partition p on [a, b] such that

$$U(f, p, \alpha) - L(f, p, \alpha) < \varepsilon$$ **(11 mks)**

QUESTION FOUR (20 MKS)

(a) Show that if the series converges when x = y then it converges for all x and x_0 is the midpoint in the interval where $|x - x_0| = |y - y_0|$ holds. **(12 mks)**

(b) Let $f \in BV[a,b]$ and let a < c < b show that V_f [a , b] = V_f [a , c] + V_f[c, b] **(8 mks)**

QUESTION FIVE (20 MARKS)

(a) Let S be the interval $[a,b]$ if the sequence $\{fn(x)\}$ of R – integrable functions on S, converges uniformly to $f(x)$ on S, show that $f(x)$ is R – integrable. **(15 mks)**

(b) Let $f(x) = \begin{cases} x^2 \cos\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

show that f is B. V. on [0, 1] if f is continuous on [0, 1] and $f^1(x)$ **(5 mks)**