KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 311**

COURSE TITLE: **REAL ANALYSIS II**

STREAM: **Y3S1**

DAY: **THURSDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **13/08/2009**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions.

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Suppose that f is continuous on [a, b] and differentiable on (a, b) show that,

(i) If $f^1(x) \geq 0 \quad \forall\ x \in (a, b)$, then f is monotonic increasing. **(5 marks)**

(ii) If $f^1(x) \leq 0 \quad \forall\ x \in (a, b)$, then is monotonic decreasing. **(5 marks)**

(b) Let $f \in BV\ [a, b]$, and consider function F defined on [a, b] by

$$F(x) = \begin{cases} V_f\ (a, x) & \text{if } a < x \leq b \\ 0 & \text{if } x = a \end{cases}$$

Show that F and F – f are both increasing. **(7 marks)**

(c) Evaluate

(i) $\int_0^3 x d(|x| - x)$ **(4 marks)**

(ii) $\int_{-2}^5 x^2\ d(|x| + [x])$ **(5 marks)**

(iii) $\int_0^\pi \cos x\ d\ (\cos x - x)$ **(4 marks)**

QUESTION TWO (20 MARKS)

(a) Define Bounded Variation. **(4 marks)**

(b) If f is continuous on [a, b] and f^1 exists and is bounded say $f^1(x) \leq A$ then show that $f \in BV\ [a, b]$. **(8 marks)**

(c) Show that if $f \in BV\ [a, b]$ then f is bounded on [a, b]. **(8 marks)**

QUESTION THREE (20 MARKS)

(a) Define total Variation. **(4 marks)**

(b) Let $f \in BV\ [a, b]$ and let $a < c < b$. Show that the total variation of the function in $BV\ [a, b]$ is $V_f\ [a, b] = V_f\ [a, c] + V_f[c, b]$ **(10 marks)**

(c) If f and g are continuous on $[a, b]$, then show that;

$$\left[\int_a^b f(x)g(x)dx\right]^2 \leq \int_a^b (f(x))^2 dx \int (g(x)^2 dx$$ **(6 marks)**

QUESTION FOUR (20 MARKS)

(a) Let P* be refinement of P, then show that $L\ (f, p, \propto)\ \leq L(f, p^*, \propto)$ **(8 marks)**

(b) Show that $\int_a^b f\ d \propto \leq \int_a^b f\ d \propto$ **(6 marks)**

(c) Evaluate;

(i) $\int_0^3 X^3\ d\ (|x|)$ **(3 marks)**

(ii) $\int_0^4 (3x - 1)\ d[X^3]$ **(3 marks)**

QUESTION FIVE (20 MARKS)

(a) Let S be the interval [a, b], if the sequence $\{f_n(x)\}$ of real valued R-intergrable functions on S converges uniformly to $f(x)$ on S then show that $\text{Lim}\int_a^b f_n\ (x)\ dx = \int_a^b \lim\ f_n(x)\ dx$

$$= \int_a^b f(x)\, dx$$

(10 marks)

(b) Prove that the more general power series about a point X_0 is given by $\sum_{n=}^{\infty} C_n(X - X_0)^n$ where X is the variable and $\sum a_n(X - X_0)^n$ converges in an interval with the mid point X_0. **(10 marks)**