KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 311**

COURSE TITLE: **REAL ANALYSIS II**

STREAM: **Y3S1**

DAY: **TUESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **22/03/2011**

INSTRUCTIONS:

- Attempt question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Given series $\sum \frac{\quad}{(\quad)}$ show that the partial sums converges to a certain limit. **(4 mks)**

(b) Define the following terms

(i) Logarithmic function **(2 mks)**

(ii) Gamma function **(2 mks)**

(c) Evaluate $\int \quad (|\ | + [\ ])$ **(3 mks)**

(d) Suppose > 0 and > 0 and $r \in \Re$ then show that

$(\quad) = \quad +$ **(4 mks)**

(e) Show that $\int \quad d \propto \leq \int \quad \propto$ **(4 mks)**

(f) Suppose and are continuous on $[\ ,\ ]$ then show that $\int \quad (\ )\ (\ )$

$\leq \int \quad (\ ) \quad \cdot \quad \int \quad (\ ) \quad .$ **(5 mks)**

(h) Let $f \in \quad [\ ,\ ]$ and defined on $[\ ,\ ]$ by F(x) = $\begin{cases} V_f(a,x) & \text{if } a < x \leq b \\ 0 & \text{if } a = x \end{cases}$

then show and $F -$ are both increasing **(5 mks)**

QUESTION TWO (20 MARKS)

(a) Suppose that is continuous on $[\ ,\ ]$ and differentiable on $(\ ,\ )$ then show that $(\quad) \geq 0$ is monotonically increasing and $(\ )$ is decreasing. **(5 mks)**

(b) (i) Define a partition and Bounded variation. **(5 mks)**

(ii) If is monotonic on $[\ ,\ ]$, then is bounded variation on $[\ ,\ ]$ **(5 mks)**

(c) Show that if $f \in \quad [\ ,\ ]$, then is bounded on $[\ ,\ ]$ **(10 mks)**

QUESTION THREE (20 MKS)

(a) Define Total variation **(2 mks)**

(b) Let $f \in$ [,] and let < < then $f \in$ [,] and $f \in$ [,]

i.e [ ], = [ ], + [ ], **(9 mks)**

(c) Define Riemman – Steitjes integral.

(d) If * is a refinement of the show that (, ∝) ≤ (, *, ∝) **(9 mks)**

QUESTION FOUR (20 MKS)

(a) Let series be defined by ∑ $(x -$) where is the variance. Show that the series converges in an interval with a point which is specific; define that point. **(10 mks)**

(b) Suppose { } is a decreasing sequence of positive terms then show that ∑ (-1) is convergent. **(10 mks)**

QUESTION FIVE (20 MARKS)

(a) Define a point-wise convergence and uniformly convergent sequences. **(4 mks)**

(b) Let be the interval [,], if the sequence { ( )} of real valued R- integrable functions on converges uniformly to ( ) on , the ( ) is R- integrable then show that

lim $_{\to}$ ∫ ( ) = ∫ lim ( ) = ∫ ( ) **(14 mks)**

(c) Show that ( ) = —— converges uniformity to ( ) = on = [0,1] **(2 mks)**