KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 221

COURSE TITLE: REAL ANALYSIS I

STREAM: Y2S2

DAY: FRIDAY

TIME: 9.00 – 11.00 A.M.

DATE: 25/03/2011

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions.

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) State and prove Cauchy's criterion for limits of functions. **(5 marks)**

(b) Show that $(\ , \ )$ contains both rational and irrational numbers. **(6 marks)**

(c) Show that if and are non-empty sets of real numbers with $= \ +$ where

$= \{ \ + \ : \ \in \qquad \in \ \}$ show that

$= \qquad +$ **(5 marks)**

(d) Explain why a function is continuous on $(\ , \ )$ but differentiate on a compact $[\ , \ ]$.

(3 marks)

(e) Prove the following properties of an ordered field

(i) $+ \ = \ + \ \Rightarrow \ =$ **(2 marks)**

(ii) $+ \ = \ , \forall \ ,$ then $= 0$ **(2 marks)**

(iii) $= 1, \ \forall \ =$ **(2 marks)**

(f) Define the following terminologies

(i) Denseness of **(2 marks)**

(ii) Supremum & infinum. **(3 marks)**

QUESTION TWO (20 MARKS)

(a) State and prove the $(\ . \ . \ )$hence show that the equation $+ \qquad - 1 = 0$ has solution $\propto$ where $0 < \propto < 1$. **(10 marks)**

(b) Prove the uniqueness of a limit in a sequence. **(4 marks)**

(c) Prove that it a sequence $\{ \ \}$ is convergent then its image is bounded. **(6 marks)**

QUESTION THREE (20 MARKS)

(a) Let F be defined on an open interval J which contains . If f is differentiable at then show that f is continuous at . **(5 marks)**

(b) (i) Define a metric space **(4 marks)**

(ii) Show that (,) = $|x -$ | is a metric space. **(6 marks)**

(iii) Let (,) be a metric space and _ then show that is open = . **(5 marks)**

QUESTION FOUR (20 MARKS)

(a) Let f be continuous on I, then the image of I under f is an interval. **(8 marks)**

(b) Let f be continuous on [,] show that f is bounded on [,]. **(7 marks)**

(c) Define a Cauchy sequence and hence show that Cauchy sequences are convergent. **(5 marks)**

QUESTION FIVE (20 MARKS)

(a) Define (i) Point-wise convergent. **(3 marks)**

(ii) Uniform convergence **(3 marks)**

(b) Let { } be a sequence of functions defined on then show that $\rightarrow$

| ( ) – ( )| < **(7 marks)**

(c) Let 0 < < 1 then the sequence { } defined by ( ) = . Show that it converges uniformly to ( ) = 0. **(7 marks)**