KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE, IN ECONOMICS AND MATHEMATICS

COURSE CODE:	**MATH 415**
COURSE TITLE:	**TESTS OF HYPOTHESIS**
STREAM:	**Y4S1**
DAY:	**MONDAY**
TIME:	**2.00-4.00 P.M.**
DATE:	**15/12/2008**

INSTRUCTIONS:

1. **Question ONE is compulsory.**
2. **Attempt question ONE and any other TWO**

PLEASE TURN OVER

QUESTION ONE-COMPULSORY (30 marks)

(a) Define the following terms

i. A Statistical test of hypothesis

ii. Power of a test

iii. Critical region

iv. A Statistical Hypothesis **(4 marks)**

(b) Define the term p-value and explain briefly how it can be used in testing a statistical hypothesis. **(2 marks)**

(c) A random sample of 10 students from a normal population showed an average weight of 66Kg and standard deviation of 10Kg . Test at 5% level of significance whether the assumption of an average weight of 70Kg for all the student population is reasonable. **(5 marks)**

(d) Let the random variable X represent the IQ scores for students taking business and mathematics, where X is normally distributed with mean μ and standard deviation 10. A random sample of 16 students was taken and the sample mean was found to be 113.5use p-value to test the hypothesis $H_0 : \mu = 110$ against $H_1 : \mu > 110$ at 10% level of significance **(4 Marks)**

(e) Let X be a random variable having a uniform distribution in the interval [-b, b]. the problem is to test the hypothesis $H_0 : b = 3$ against $H_1: b > 3$ the critical region is given by Cr = { x : $|x| > 2.99$ find

i. The probability of type I error

ii. The power of the test at $b = {}^{7}/_{2}$ **(8marks)**

(f) Let $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Test the hypothesis $H_0 : \mu = \mu_0$ against $H_1: \mu < \mu_0$ assuming that σ^2 is known i.e $\sigma^2 = \sigma_0^2$ and the level of significance is α **(7marks)**

QUESTION TWO (20 Marks)

(a) Define the uniformly most powerful test. **(3 marks)**

(b) Let $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ be a random sample from a variable X where $X \sim N(0, \sigma^2)$ Obtain a uniformly most powerful size α test for $H_0 : \sigma = \sigma_0$ verses $H_1: \sigma > \sigma_0$ **(10 marks)**

(c) A medical officer asserts that the mean sleep of young babies is 14 hours a day. A random sample of 64 babies was taken from a normal population with mean μ and unknown variance σ^2 the results shows that their mean sleep was only 13 hours 20 minutes with a standard deviation of 3 hours. At 5 % level of significance, test the assertion that the mean sleep of babies is less than 14 hours a day. Hence or otherwise compute the p – value of the test. **(7 marks)**

QUESTION THREE (20 marks)

(a) State and prove the Neyman – Pearson Lemma for testing a simple hypothesis versus a simple alternative. **(13 marks)**

(b) Use the Neyman – Pearson Lemma to determine the nature of best critical region for testing hypothesis $H_0 : \mu - \mu_0 = 0$ against $H_1: \mu - \mu_0 > 0$ if the p. d. f. is given as

$$f(x,\mu)=\begin{cases}(1-\mu)x^{\mu}\text{..........}\mu > 0\text{..........}0\le x\le 1\\ 0\text{....}elsewhere\end{cases}$$ **(7 marks)**

QUESTION FOUR (20 Marks)

a) Consider a sample linear model $y=\alpha+\beta x+$ ª, where α and β are unknown constants $\ell \sim N(0, \sigma^2)$ and assume that $Y \sim N(\alpha+\beta x, \sigma^2)$. The hypothesis to be tested is $H_0 : \beta = 0$ against $H_1: \beta > 0$ at γ level of significance.

i. Derive the likelihood ratio test for the above hypothesis and show that it can be based on the sample correlation coefficient ℓ where

$$\ell=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

ii. Derive the test statistics for the above hypothesis **(16 marks)**

b) For the above linear model the following summary statistics ere obtained $\bar{x}=1.4$, $\sum x = 19.656$, $\bar{y}=76.67$, $\sum y^2 = 58846.09$, $\sum xy = 1074.802$, n = 10 test the hypothesis $H_0 : \beta = 0$ against $H_1: \beta > 0$ at $\gamma = 0.025$ level of significance

(4 marks)

QUESTION FIVE (20 Marks)

a) Suppose the random variable W and Z have a joint distribution given by

$$f(w,z)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-ª^2}}.\exp\frac{-1}{2(1-ª^2)}\left[\frac{(w-u_1)^2}{\sigma_1^2}-2\frac{(e-u_1)}{\sigma_1}.\frac{(z-u_2)}{\sigma_2}ª+\frac{(z-u_2)^2}{\sigma_2^2}\right]$$ test the independence of he random variables W and Z. **(11 marks)**

b) In an experiment to investigate the relationship between obesity and high blood pressure condition, the girths of ten men and their corresponding blood pressure were measured as shown in the table below

Girths (W)	40	39	33	44	42	50	31	44	41	38
Pressure (Z)	142	123	120	184	165	190	121	140	130	121

Test the hypothesis that the size of the girth is independent of the blood pressure at $\alpha = 0.01$ level of significance. **(9 marks)**