

KABARAK



UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF SCIENCE IN
ECONOMICS AND MATHEMATICS**

COURSE CODE: MATH 416

COURSE TITLE: TIME SERIES

STREAM: Y4S1

DAY: WEDNESDAY

TIME: 2.00 – 4.00 P.M.

DATE: 18/03/2009

INSTRUCTIONS:

Attempt **QUESTION ONE** and **ANY OTHER TWO** questions.

PLEASE TURN OVER

Question One (30mks)

(a) The data below shows the sales made by a company from 1997 to 2000.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1997	50	35	10	28
1998	54	38	12	34
1999	58	41	15	41
2000	62	44	18	46

- (i) Obtain a 4 moving average trend. **(3mks)**
- (ii) Using an appropriate model, obtain seasonal and random variations. **(4mks)**
- (iii) Hence forecast for 2001 quarter 1, quarter 2. **(2mks)**

(b) Calculate the autocovariance and hence the autocorrelation function (ACF) of the data below.

1	2	3	4	5	6	7	8
50	40	32	24	16	13	9	6

Hence plot the correlogram **(6mks)**

(c) Determine whether the following ARMA models are stationary

- (i) $\phi = 1 - 0.24B + \dots$
- (ii) $\phi = 1 - 0.9B - 0.2B^2 + \dots$
- (iii) $\phi = 1 - 0.8B + 0.48B^2 - \dots$ **(4mks)**

(d) Determine the invertibility and ACF of the following MA (2) process

- (i) $\theta = 1 - 0.36B - 0.36B^2$
- (ii) $\theta = 1 + 0.6B - 0.5B^2 + \dots$ **(4mks)**

(e) Obtain the spectral density of an AR(1) process given by

$\phi = 1 - \phi_1 B + \dots$ **(5mks)**

Question Two (20mks)

- (a) (i) State Wold decomposition theorem. **(4mks)**
(ii) Hence derive the Auto regression model AR(1) and AR (2). **(2mks)**
- (b) Given ARMA (1, 1) model obtain
(i) the ACF **(4mks)**
(ii) Explain the behavior of the ACF **(2mks)**
- (c) Calculate the ACF for $\phi = 0.5$ $\theta = 0.3$ $\rho = 0.5$
Hence plot it. **(8mks)**

Question Three (20mks)

- (a) Explain the following terms:
(i) invertibility
(ii) weekly stationary
(iii) causality
(iv) correlogram **(8mks)**
- (b) Compute the spectral density of the filtered process
$$S(\omega) = -[\dots + \dots + \dots]$$
 (6mks)
- (c) Given the series
- | Year | Q1 | Q2 | Q3 | Q4 |
|------|----|----|----|----|
| 1 | 59 | 25 | 75 | 12 |
| 2 | 63 | 28 | 79 | 17 |
| 3 | 70 | 32 | 84 | 22 |
| 4 | 77 | 36 | 89 | 27 |
- (i) plot the time series
(ii) obtain the trend
(iii) forecast for year 5 **(6mks)**

Question Four (20mks)

(a) Briefly explain the factors that make a time series non-stationary. **(8mks)**

(b) The spectral density of a real valued time series $\{X_t\}$ is defined on $[0, \pi]$ by

$$f(\omega) = \begin{cases} 100 & \text{if } \pi/6 - \epsilon < \omega < \pi/6 + 0.01 \\ 0 & \text{otherwise} \end{cases}$$

And on $[-\pi, 0]$ by $f(\omega) = f(-\omega)$

(i) Evaluate the ACF of $\{X_t\}$ at lag 0 and 1 **(6mks)**

(ii) What is the variance of $\sum_{t=1}^{12} X_t$ **(6mks)**

Question Five (20mks)

(a) Determine the discrete fourier transform of the following series

(i) $X_t = 0.5^t + \dots$

(ii) $X_t = 0.3^t + 0.2^t + \dots$ **(8mks)**

(b) Find the ACF and PACF of the series $X_t = 0.5^t + 0.2^t + \dots$ **(8mks)**

is the series stationary. **(4mks)**