# KABARAK UNIVERSITY

## EXAMINATIONS

## 2008/2009 ACADEMIC YEAR

## FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

**COURSE CODE:** **MATH 222**

**COURSE TITLE:** **VECTOR ANALYSIS**

**STREAM:** **Y2S2**

**DAY:** **TUESDAY**

**TIME:** **9.00 – 11.00 A.M.**

**DATE:** **24/03/2009**

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**INSTRUCTIONS:**

(i) Attempt **QUESTION ONE** and **ANY OTHER TWO** questions.
(ii) Show all your working and be neat

**PLEASE TURN OVER**

**<u>Question One</u> (30 Marks)**

a) (i) Define the terms scalar, vector and unit vector. (3 marks)

(ii) Find the unit vector in the direction of the vector $\vec{\ } - 2\vec{\ }$ where

$\vec{\ } = 2\hat{\ } + 2\hat{\ } -$ and $\vec{\ } = 3 + \hat{\ } - 5$ . (3 marks)

b) If $\vec{\ } = \frac{\hat{\ }\ \hat{\ }}{\ }$, find $\nabla \cdot \vec{\ }$. (4 Mark)

c) If the three sides of a triangle are of length $\vec{\ }$, $\vec{\ }$ and $\vec{\ }$ and if the angle opposite the side of length $\vec{\ }$ is , show that $\vec{\ } = \vec{\ } + \vec{\ } - 2\vec{\ }\vec{\ }\cos$ . (5 marks)

d) If $\vec{\ }$ and $\vec{\ }$ are two nonzero, non-parallel vectors and $\vec{\ }$ any vector in the plane of $\vec{\ }$ and $\vec{\ }$, then show that $\vec{\ }$ can be expressed as a linear combination of $\vec{\ }$ and $\vec{\ }$ i.e. $\vec{\ } = \sigma\vec{A} + \alpha\vec{B}$ where σ and α are uniquely determined scalars. (6 marks)

e) If $= 3$ , $=$ $- 2$ then evaluate $\nabla(\nabla u \cdot \nabla\ )$. (6 Marks)

f) If $\vec{\ } = (3\ \ y - \ )\hat{\ } + (\ \ -\ \ \sin)\hat{\ } + \ \ \sin$ , find $\frac{\vec{\ }}{\ }$ (3 marks)

**<u>Question Two</u> (20 Marks)**

a) (i) Find the normal and binormal vectors for the curve

$(\ ) = \ ,\ (\ ) = 3\sin\ ,\ (\ ) = 3\cos$ . (9 Marks)

(ii) Determine the curvature for the curve in (i) above. (2 Marks)

b) Evaluate $\iint \vec{\ } \cdot \vec{\ }$ where $\vec{\ } = 4\ \hat{\ } - \ \hat{\ } +$ and is the surface of the cube bounded by $= 0, = 1, = 0, = 1, = 0, = 1$ (6 Marks)

c) State Gauss Divergence theorem in word and give its mathematical equivalence. (3 marks)

**<u>Question Three</u> (20 Marks)**

a) Prove that $(3\ + 2\ + 4\ )\hat{\ } + (2\ + 5\ + 4\ )\hat{\ } + (4\ + 4\ - 8\ )$ is both solenoidal and irrotational. (6 Marks)

b) Evaluate $\frac{\vec{\ }}{\ } \times \frac{\vec{\ }}{\ }$ at the point $(1,1,0)$ if $\vec{\ } = \ \hat{\ } + \ \hat{\ } -$ and $\vec{\ } = \ \hat{\ } - \ \hat{\ } +$ . (4 Marks)

c) Derive the scale factors for a cylindrical coordinate system; hence prove that this coordinate system is orthogonal. (10 Marks)

**Question Four (20 Marks)**

a) Prove that $\nabla \times \emptyset \vec{\ } = \nabla\emptyset \times \emptyset \ \nabla \times \vec{\ }$. (6 Marks)

b) Show that the limit of the sum of two vector functions is the sum of their limits, i.e. if

$\lim_{t\to} \vec{\ }(\ ) = \vec{\ }$ and $\lim_{t\to} \vec{\ }(\ ) = \vec{\ }$

then $\lim_{t\to} \vec{\ }(\ ) + \vec{\ }(\ ) = \vec{\ } + \vec{\ }$ (6 Marks)

c) If $=\ \mp$ and $= 2\ y -$ find: (8 Marks)

(i) $\nabla(\ \ +)$

(ii) $\nabla(\ \ )$ at the point $(1,0,-2)$.

**Question Five (20 Marks)**

a) Use Stoke's theorem to evaluate $\int (\ \ + \ \ )$ when is the square on the -plane with vertices $(1,0)$, $(-1,0)$, $(0,1)$, $(0,-1)$. (7 Marks)

b) A particle moves along the curve $= 2\ ,\ = \ -4\ ,\ 3\ -5$ where the time is. Find the components of its velocity and acceleration at time $= 1$ in the direction $\hat{\ } - 3\hat{\ } + 2$ . (3 Marks)

c) (a) Show that $\vec{\ } = (2\ \ +)\hat{\ } + \ \hat{\ } + 3$ is a conservative field. (3 Marks)

(b) Find the work done in moving an object from $(1,-2,1)$ to $(3,1,4)$. (4 Marks)