KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 222

COURSE TITLE: VECTORS ANALYSIS

STREAM: SESSION V

DAY: FRIDAY

TIME: 9.00 – 11.00 A.M.

DATE: 15/04/2011

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE (30MKS) COMPULSORY

(a) If **r** = t^2i-tj + (2t +1)k. Find at t = 0 the value of

- (i) |d**r**/dt| (3mks)
- (ii) $|d^2\mathbf{r}/dt^2|$ (2mks)

(b) If **r** = $5t^2$i + tj-t^3k and **s** = sin tj - cos tj find the value of

- (i) d/dt(**r**.**s**) (3mks)
- (ii) d/dt(**r**×**s**) (4mks)
- (iii) d/dt(**r**.**r**) (3mks)

(c) If F = x^2j + y^2j + Z^2k. Find

- (i)∇.F (3mks)
- (ii)∇×F (3mks)

(d) Determine the total work done in moving a particle in a force field given by F = 3xyi - 5tj + 10xk along the curve x = t^2 +1, y = $2t^2$, t = t^3 from t = 1 to t = 2 (4mks)

(e) Show that

d/dt(U + V) = du/dt + dv/dt for any two vectors U and V (2mks)

(f) Given that r = a costi + a sin tj + btk, Show that $(r)^2 = a^2 + b^2$ (3mks)

QUESTION TWO (20MKS)

(a) Given that r = a costi + a sintj + at tan k Find

- (i) $|dr/dt \times d^2r/dt^2|$ (5mks)
- (ii) $dr/dt \bullet d^2r/dt^3 \times d^3r/dt^3$ (5mks)

(b) Given that r = a costi + a sin tj + btk show that $(r' \times r'')^2 = a^2(a^2+b^2)$ (5mks)

(c) Determine the values of a,b,c,d for which the vector r=(x+2y+az)I + (bx-3y-z)j + (4x + cy + 2z) is irrotational (3mks)

(d) Determine the constant a so that the vector V= (x + 3y)i + (y - 2z)j + (x + az)k is solenoidal. (2mks)

QUESTION THREE (20MKS)

(a) The acceleration at any instant t of a moving particle is given by $d^2r/dt^2 = 12 \cos 2t i - 8 \sin 2t j + 16t k$.

Find the velocity V and displacement r at any time t if at t = o, it is known that v = o and r = o. (6mks)

(b) Determine r at any instant t if d^2r/dt^2 $6ti - 24t^2j + 4\sin t k$ .Given that $r = 2i + j$ and $dv/dt = -i-3k$ at t = o

(4mks)

(c) If $A = (3x^2 + 6y)i - 14ytj + 20xt^2k$. Evaluate $\int$. dr from (0,0,0) to (1,1,1) along the following path C:

- (i) $x = t,\ y = t^2,\ t = t^3$ (5mks)
- (ii) The straight lines (0,0,0) to (1,0,0) then to (1,1,0) and then to (1,1,1) (5mks)

<u>QUESTION FOUR (20MKS)</u>

(a) Let $A = 2yzi - x^2yj + xt^2k$, $B = x^2i + ytj - xyk$ and $Ø = 2x^2 yt^2$ find

- (i) $(A.\nabla)Ø$ (3mks)
- (ii) $(A\times\nabla)Ø$ (4mks)
- (iii) $A\times\nabla\ Ø$ (4mks)

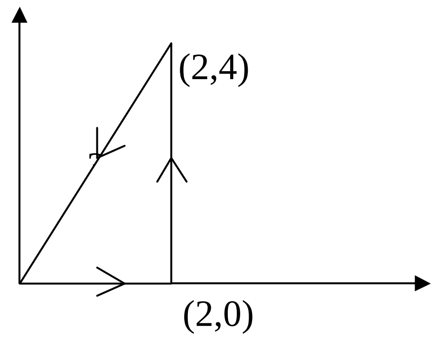
(b) A particle moves so that its position vector is given by $r = \cos ti + \sin btj$ where b is a constant. Show that

- (i)The velocity of the particle is perpendicular to r (4mks)
- (ii)The acceleration a is directed towards the origin and has magnitude proportional to the distance from the origin. (5mks)

<u>QUESTION FIVE (20MKS)</u>

(a) Determine the circulation of the vector function $V=2yi+xj$ around the curve shown below

- (i) Direct integration (5mks)
- (ii) Use stokes Theorem (5mks)



(b) (i) State the divergence theorem. (2mks)

(ii) In the divergence theorem, let the vector function U=Øi, where Ø is a scalar function. Derive the resulting integral theorem. (8mks)