KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 222**

COURSE TITLE: **VECTOR ANALYSIS**

STREAM: **SESSION IV**

DAY: **THURSDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **12/08/2010**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** questions

PLEASE TURNOVER

QUESTION ONE: 30 MARKS

a) Find the unit vector perpendicular to the plane of the vector $\vec{A} = 3i - 2j + 4k$ and $\vec{B} = i + j - 2k$. (4 marks)

b) Find the directional derivative of $\phi = xyz$ at (1,2,3) in the direction from (1,2,3) to (1,-1,-3). (6 marks)

c) A particle moves along the space curve $r(t) = (t^3 + 2t)i - 3e^{-2t}j + 2\sin 5tk$.find the

(i) Velocity

(ii) Acceleration at any time (4 marks)

d) Find the volume of a parallelepiped with the sides $\vec{a} = 3i - j, \vec{b} = j = 2k, \vec{c} = i + 5j + k$. (5 marks)

e) Verify that Divcurl $\phi = 0$ (6 marks)

f) Find the curvature of the twisted cube $\vec{r}(t) = < t, t^2, t^3 >$ at a point (0,0,0) (5 marks)

QUESTION TWO: 20 MARKS

a) If $\vec{A} = xi - x^2 j + (x-1)k$ and $\vec{B} = 2x^2 i + 6xk$ evaluate (i) $\int_0^2 \vec{A}.\vec{B}dx$ (ii) $\int_0^2 \vec{A} \times \vec{B}dx$ (8 marks)

b) Find the value of λ if $\vec{A} = \cos \lambda xi + \sin \lambda xj$ satisfies the differential equation $\frac{d^2\vec{A}}{dx^2} = -9\vec{A}$ (6 marks)

c) Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$ (6 marks)

QUESTION THREE: 20 MARKS

a) Given the space curve $x = 3\cos t, y = 2\sin t$ find the curvature of the ellipse at the points corresponding to $t = 0$ and $t = \frac{\pi}{2}$. (7 marks)

b) If $\vec{R} = x^2 yi - 2y^2 zj + xy^2 z^2 k$, find $\left| \frac{\partial^2 \vec{R}}{\partial x^2} \times \frac{\partial^2 \vec{R}}{\partial y^2} \right|$ at the point (2,1,-2)

c) Show that the vectors $\vec{a} = i + 4j - 7k, \vec{b} = 2i - j + 4k$ and $\vec{c} = -9j + 18k$ are coplanar. (5 marks)

QUESTION FOUR: 20 MARKS

a) Find the volume of the parallelepiped of sides $\vec{a} = i - j + 2k, \vec{b} = 2i + j + 4k$ and $\vec{c} = -i - 2j - 9$. (4 marks)

b) If $\vec{a} = x^2 i - yj + xzk, \vec{b} = yi - xj + xyzk$ and $\vec{c} = i - yj + x^3 zk$ evaluate at (1,-1,2)

(i) $\frac{\partial^2}{\partial x \partial y}(\vec{a} \times \vec{b})$ (ii) $\frac{\partial \vec{F}}{\partial x}$ given that $\vec{F} = \vec{a}.(\vec{b} \times \vec{c})$ (10 marks)

c) Evaluate div $\vec{F} = x^2 yzi + 3xyz^3 j + (x^2 - z^2)k$ (6 marks)

QUESTION FIVE: 20 MARKS

a) Verify green's theorem in the plane for $\int_c 4x^2 ydy + 2ydy$ where c is the closed curve is a triangle with vertices (0,0), (1,2) and (0,2). (7 marks)

b) Evaluate $\iint_s \vec{F}.nds$ where $\vec{F} = x^2 yi + 2xzj + yz^3 k$ and s is the surface of the rectangular solid determined by $0 \le x \le 1.0 \le y \le 2, 0 \le z \le 3$. (7 marks)

c) Find the equation of the tangent plane and the normal line to the surface $x^2 + y^2 + 2z^2 = 23$ at (1,2,3). (6 marks)