KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 222**

COURSE TITLE: **VECTOR ANALYSIS**

STREAM: **SESSION IV**

DAY: **SATURDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **10/04/2010**

INSTRUCTIONS:

Answer Question **ONE** and any other **TWO** Questions.

PLEASE TURN OVER

QUESTION ONE: 30 MARKS

a). Let $\tilde{A} = (2x^2y - x^4)\hat{\imath} + (e^{xy} - y\,sinx)\hat{\jmath} + (x^2cosy)\hat{k}$

show that $\frac{\partial^2 \tilde{A}}{\partial x \partial y} = \frac{\partial^2 \tilde{A}}{\partial y \partial x}$ [4 marks]

b). Prove that the diagonals of a parallelogram bisect each other [8 marks]

c). Find the volume of the Tetrahedron with sides $\tilde{a}, \tilde{b}, and\ \tilde{c}$ given by $\tilde{a} = \tilde{\imath} + 2\tilde{k}$,

$\tilde{b} = 4\tilde{\imath} + 6\tilde{\jmath} + 2\tilde{k}$ and $\tilde{c} = 3\tilde{\imath} + 3\tilde{\jmath} - 6\tilde{k}$ [6 marks]

d). Evaluate the iterated integral

$$\int_1^3 \int_{\frac{1}{2}x}^{4x} 2xy\, dydx$$ [5 marks]

e). State Stoke's Theory and use it to evaluate $\oint_c \tilde{A}.dr$ where $\tilde{A}$ = (y-z+2)i + (yz+4)j – xzk and C is the boundary of the region y=0, x=0, z=0 y=2, x=2, z=2 above the xy plane. [7 marks]

QUESTION TWO: 20 MARKS

Sketch the space curve with parametric equations

$x = 3\ \cos t,\ \ y = 3\ \sin t$ and $z = 4t$. Find;

(i) The unit tangent T
(ii) The principal normal N
(iii) Curvature K
(iv) Radius of curvature P
(v) The binormial B
(vi) Torsion T and
(vii) Radius of torsion S [20 marks]

QUESTION THREE: 20 MARKS

a). State the first form of the divergence theorem of Gauss. [4 marks]

b). Verify the divergence theorem in (a), above where

$\tilde{A} = 2x\hat{\imath} + zy\hat{\jmath} + x\hat{k}$ and S is the surface of the tetrahedron bounded by the planes

$x = 0,\ y = 0, z = 0$ and $x + y + z = 1$ [12 marks]

c). Find the angle between the vectors

$\tilde{A} = 3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$ and $\bar{B} = 4\hat{\imath} - 3\hat{\jmath} + \hat{k}$ [4 marks]

QUESTION FOUR: 20 MARKS

a). Prove that the vectors $\bar{A} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, $\tilde{B} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\tilde{C} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ can form the sides of a right angled triangle. Find the area of this triangle [5 marks]

b). Find the equation of the plane through $P(2,-1,1)$, $Q(3,2,-1), R(-1,3,1)$ in the form

$ax + bytcz + d = 0$ [6 marks]

c). Show that the vector field $\tilde{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational and find its scalar potential. [9 marks]

QUESTION FIVE: 20 MARKS

a). Evaluate $\iint_S \tilde{A} \cdot nds$ where $\tilde{A} = 2y\hat{\imath} - z\hat{\jmath} + x^2\hat{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$ and $z = 6$ [10 marks]

b). A particle moves along a curve whose parametric equations are

$x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time.

(a) Determine its velocity and acceleration at any time

(b) Find the magnitude of the velocity and acceleration at t = 0 [10 marks]

QUESTION SIX: 20 MARKS

a). Find the directional derivative of the function $Q = x^3y + 3y^2z - 4xz^2$ at $(3,1,-1)$ in the direction of $i + 4\hat{\jmath} - 3\hat{k}$ [6 marks]

b). Evaluate $\nabla^2\left(div\left(\frac{\tilde{r}}{r^2}\right)\right)$ where $\tilde{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ [8 marks]

c). Show that if $\tilde{A}$ is a vector-valued function and $\emptyset$ is a scalar function then

$div\left(\emptyset\tilde{A}\right) = grad\,\emptyset \cdot \tilde{A} + \emptyset\, div\,\tilde{A}$ [6 marks]