KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 222

COURSE TITLE: REAL ANALYSIS I

STREAM: SESSION III

DAY: TUESDAY

TIME: 2.00 – 4.00 P.M.

DATE: 01/12/2009

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) The position vector of a moving particle is $R(t) = 5 \sin 4t\hat{\imath} + 5\cos 4t\hat{\jmath} + 10t\hat{k}$. Find the magnitude of the velocity and acceleration at time $t = 0$ **(4 marks)**

(b) The acceleration of a particle at any time $t \geq 0$ is given by $a = e^{t}\hat{\imath} + e^{2t}\hat{\jmath} + \hat{k}$. If at $t = 0$, the displacement is $r = o$ and the velocity is $v = \hat{\imath} + \hat{\jmath}$, find r and v at any time t. **(6 marks)**

(c) If $\emptyset = x^2 + y^2 + z^2$ and $A = xy^2\hat{\imath} + yz^2\hat{\jmath} + x^2z\hat{k}$, Find

(i) Grad $\emptyset$ **(2 marks)**

(ii) Div A **(2 marks)**

(iii) Curl A **(4 marks)**

(d) Find the equation of the tangent plane to the surface

$$F\ (x_1\ y_1\ z) = x^3 + 3xyz + 2y^3 - z^3 - 5 \text{ at the point } (1,1,1)$$

(4 marks)

(e) Evaluate the line integral

$\int_c (x^2 - 2y^3)\,dy + (x + 5y)dy$ a long a straight line from $(1,1)$ to $(2,2)$ **(4 marks)**

(f) Show that $F = (2xy + 3)\hat{\imath} + (x^2 - 4z)\hat{\jmath} - 4y\hat{k}$ is a conservative force field. **(4 marks)**

QUESTION TWO (20 MARKS)

(a) Evaluation $\iint_s x\,ds$ where S is that portion of the surface $y = 2x^2 - z$ in the first octant bounded by $x = 0, x = 2, y = 4$ and $y = 8$ **(8 marks)**

(b) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = a^2$ **(4 marks)**

(c) Given $f(x, y, z) = x^2\,y\hat{\imath} + z\hat{\jmath} - (x + y - z)\hat{k}$ Find

(i) $\nabla \cdot F$ **(2 marks)**

(ii) ∇ x F **(3 marks)**

(iii) $\nabla(\nabla \cdot F)$ **(3 marks)**

QUESTION THREE (20 MARKS)

(a) Given that $\underline{R} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\underline{R}|$, prove that $\nabla r^n = nr^{n-2}\,R$ **(10 marks)**

(b) Show that the vector field $\underline{F} = 2xye^z\hat{\imath} + (x^2e^z + y)\hat{\jmath} + (x^2ye^z - z)\hat{k}$ is a gradient field and find its scalar potential function. **(10 marks)**

QUESTION FOUR (20 MARKS)

(a) Verify Green's theorem in the plane for $\oint_c (x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy,$ where C is a square with vertices at $(0,0), (2,0), (2,2)$ and $(0,2)$ **(10 marks)**

(b) Show that the area bounded by a simple closed curve is given by $\frac{1}{2}\oint_c \; xdy - ydx$

(3 marks)

(c) If $\underline{A} = (3x^2 - 6yz)\hat{\imath} + (2y + 3\,xz)\hat{\jmath} + (1 - 4xyz^2)\hat{k},$ evaluate $\int_c \; A \cdot dr$

(i) From $(0,0,0)\ to\ (1,1,1)$ a long the path $x = t,\ y = t^2,\ z = t^3$ **(4 marks)**

(ii) Straight line from $(0,1,1)\ to\ (1,1,1)$ **(3 marks)**

QUESTION FIVE (20 MARKS)

(a) Evaluate $\iint_s \; A \cdot nds$ where $\underline{A} = xy\hat{\imath} - x^2\hat{\jmath} + (x + z)\hat{k}$ S is that portion of the plane $2x + 2y + z = 6$ included in the first octant and n is a unit normal to S. **(14 marks)**

(b) Use divergence theorem to evaluate $\iint_s \; A \cdot nds$ where $\underline{A} = (2x - z)\hat{\imath} + x^2y\hat{\jmath} - z^2\hat{k}$ Taken over the region bounded by $x = 0,\ \ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$

(6 marks)