KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 222

COURSE TITLE: VECTOR ANALYSIS

STREAM: Y2S2

DAY: THURSDAY

TIME: 2.00 – 4.00 P.M.

DATE: 17/03/2011

INSTRUCTIONS:

1.Question **ONE** is compulsory.

2. Attempt question ONE and any other TWO

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

a) A particle moves so that its position vector is given by $\underline{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$ where ω is a constant **(5 marks)**

i) Show that the velocity $\underline{v}$ of the particle is perpendicular to $\underline{r}$

ii) The acceleration $\underline{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin.

b) Given that $\vec{A} = 2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}$ and $\phi = 2x^2yz^3$. Find $(\vec{A}\times\vec{\nabla})\phi$ **(5 marks)**

c) Determine the constant a so that the vector $\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is a solenoid **(2 marks)**

d) The acceleration of a particle at any time $t \geq 0$ is given by $\underset{\sim}{a} = 12\cos 2t\hat{i} - 8\sin 2t\hat{j} + 16t\hat{k}$. If at $t = 0$, the displacement is $r = o$ and the velocity is $v = i + j$, find r and v a any time t. **(6 marks)**

e) Given that $A = (3x^2 + +6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$ evaluate $\int_C \vec{A}\bullet d\underset{\sim}{r}$ from $(0,0,0)$ to $(1,1,1)$ along the path $x = t$, $y = t^2$ $z = t^3$ **(4 marks)**

f) Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ **(3 marks)**

g) Verify green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve bounded by $y = x$ and $y = x^2$ **(3 marks)**

h) Express curl $\vec{A}$ in orthogonal coordinates. **(3 marks)**

QUESTION TWO (20 MARKS)

(a) Given that A and B are differentiable functions of a scalar u show that **(5 marks)**

$$\frac{d}{du}A\bullet B = A\bullet\frac{dB}{du} + B\bullet\frac{dA}{du}$$

(b) Find the unit tangent vector to any point on the curve $x = t^2 + 1$, $y = 4t - 3$ and $z = 2t^2 - 6t$ hence find the unit tangent at the point where t = 2 **(5 marks)**

(c) A space curve is given by the equations $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$ find **(10 marks)**

i) the curvature κ

ii) The torsion τ .

QUESTION THREE (20 MARKS)

a) Show that $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\vec{\nabla}^2 \vec{A} + \vec{\nabla}(\vec{\nabla} \bullet \vec{A})$ **(7 marks)**

b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2,-1,2)$ **(5 marks)**

c) Given a vector $\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)$ find **(8 marks)**

i) Find the values of *a, b, c* so that the vector is irrotational

ii) Express the vector as a gradient of a scalar function

QUESTION FOUR (20 MARKS)

a) Given that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ **(9 marks)**

i) Show that $\vec{F}$ is a conservative force field.

ii) Find the scalar potential

iii) Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$

(b) Evaluate $\iint_S (\overset{\circ}{\nabla} \times \overset{\circ}{F}) \bullet \hat{n}dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the x-y plane given that $F = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$ **(11 marks)**

QUESTION FIVE (20 MARKS)

a) Use divergence theorem to evaluate $\iint_s A \cdot n ds$ where $\underline{A} = (2x - z)\hat{i} + x^2y\hat{j} - z^2\hat{k}$ taken over the region bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$ **(8 marks)**

b) Verify Stokes theorem for $\vec{A}(2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **(6 marks)**

c) Find the square of the element of arc length in cylindrical coordinates and determine the corresponding scale factors. **(6 marks)**