KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 222

COURSE TITLE: VECTOR ANALYSIS

STREAM: SESSION III

DAY: TUESDAY

TIME: 9.00 – 11.00 A.M

DATE: 30/11/2010

INSTRUCTIONS:

PLEASE TURN OVER

QUESTION ONE (30 MARKS) COMPULSORY

(a) i) Determine the value of a so that

A = **i** + a**j** + **k** and **B** = 4**i** – 2**j** – 2**k** are perpendicular. (2mks)

ii) Given that **A** = 3**i** – **j** + 2**k** and **B** = 2**i** + **j** – **k** and **C**= **i** – 2**j** + 2**k**. Find (A x B) xC and A x (B x C) hence show that (A x B) x C ≠ A x (B x C) (4mks)

(b) i) If A and B are differentiable functions of a scalar u, show that (3mks)

$\frac{d}{du}(\mathbf{A}\cdot B) = \mathbf{A}\cdot\frac{dB}{du} + \frac{dA}{du}\cdot\mathbf{B}$

ii) Find the velocity and acceleration of a particle which moves along the curve x = 2sin3t, y = 2cos3t, z = 8t at any time t > 0, hence determine the magnitude of the velocity and acceleration. (4mks)

(c) If $\emptyset(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\emptyset$ at the point (1, -2, -1) (4mks)

(d) Define curl A $=\nabla \times a$, hence find curl A at the point (1,-1, 1) given that $A = xz^3\mathbf{i} - 2x^2yz\mathbf{j} + 2yz^4\mathbf{k}$. 4mks)

(e) If $A = 5t^2\mathbf{i} + t\mathbf{j} - t^3\mathbf{k}$ and **B** = sin **i** – cost **j**, find:

i) $\frac{d}{dt}(\mathbf{A} \bullet \mathbf{B})$ (2mks)

ii) $\frac{d}{dt}(\mathbf{A} \times \mathbf{B})$ (3mks)

f) (Find the total work done in moving a particle in a force field given by $F = 3xy\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}$ along with curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t = 1 to t =2 (4mks)

QUESTION TWO (20 MARKS)

a) If $\mathbf{A} = x^2yi - 2xz\mathbf{j} + 2yz\mathbf{k}$ find curl curl A. (4mks)

b) Let **A** = t**i** – 3**j** + 2t**k**, **B** = **i** – 2**j** + 2**k**, C = 3**i** + t**j** – **k**

Evaluate i) $\int_1^2 A \cdot (B \times C)dt$ (5mks)

ii) $\int_1^2 A \times (B \times C)dt$ (5mks)

c) Evaluate $\iint_s F \cdot nds$ where $F = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$ and s is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1 (6mks)

QUESTION THREE (20 MKS)

a) Verify stokes theorem for $A = (2x-y)\,\mathbf{i} - yz^2\,\mathbf{j} - y^2z\mathbf{k}$ where s is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (8mks)

b) If $A = x^2z\mathbf{i} + yz^2\mathbf{j} - 3xy\mathbf{k}$ and $B = y^2\mathbf{i} - yz\mathbf{j} + 2x\mathbf{k}$, find $(A.\,\nabla)B$ (4mks)

c) The acceleration of a particle at any time $t \geq o$ is given by

$a = \frac{dv}{dt} = 12\cos 2t\mathbf{i} - 8\sin 2t\mathbf{j} + 16t\mathbf{k}$, if the velocity v and displacement r are zero at t = o, find v and r at any time. (8mks)

QUESTION FOUR (20 MKS)

a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. (7mks)

b) Prove $\nabla.(A + B) = \nabla.A + \nabla.B$ (5mks)

c) Verify Greens theorem in the plane for $\oint_C (xy + y^2)\,dx + x^2dy$, Where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (8mks)

QUESTION FIVE (20 MKS)

a) Show that $\nabla \times (\nabla^2 u) = \nabla^2 (\nabla \times u)$ (4mks)

b) Evaluate:

i) Grad (div F)

ii) Div (Curl F) for the vector field $F = yz\mathbf{j} + zx\mathbf{k}$ (3mks)

c) Use Greens Theorem to evaluate $\oint_c (x^2 + 2y)\,dx + (x + 3y^2)\,dy$ (3mks)

d) Verify the divergence theorem for $A = 4x\mathbf{i} - 2y^2 j + z^2 k$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 8$ (10mks)