

**KABARAK**



**UNIVERSITY**

**EXAMINATIONS**

**2008/2009 ACADEMIC YEAR**

**FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE**

**COURSE CODE: MATH 222**

**COURSE TITLE: VECTOR ANALYSIS**

**STREAM: SESSION VI**

**DAY: MONDAY**

**TIME: 2.00 – 4.00 P.M.**

**DATE: 06/04/2009**

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**INSTRUCTIONS:**

- Attempt **QUESTION ONE** and **ANY OTHER TWO** questions.

**PLEASE TURN OVER**

**Question One (30 Marks)**

- (a) If  $\vec{r} = x\hat{i} + (3x - 2y)\hat{j} - (3x + 2y)\hat{k}$ , find  $\frac{\delta \vec{r}}{\delta x \delta y}$  (3 marks)
- (b) For what values of P are the vectors  $\vec{r} = x\hat{i} - 2y\hat{j} + z\hat{k}$  and  $\vec{s} = 2x\hat{i} + y\hat{j} - 4z\hat{k}$  perpendicular? (3 marks)
- (c) Evaluate  $(2x - 4y) \times (x \times 2y)$  (2 marks)
- (d) Find the unit tangent vector for a curve whose parametric equations are  $x = t$ ,  $y = 2 \cos 3t$ ,  $z = 2 \sin 3t$  at  $t = 0$  (5 Marks)
- (e) A particle moves along a curve whose parametric equations are  $x = t$ ,  $y = \cos t$ ,  $z = t$ . Determine the velocity and acceleration at time  $t = 0$  (5 marks)
- (f) Evaluate  $\text{div } (2x\hat{i} - y\hat{j} + 3z\hat{k})$  (3 marks)
- (g) If  $\vec{r} = (5x - 6y)\hat{i} + (2x - 4y)\hat{j}$ , evaluate  $\int_C \vec{r} \cdot d\vec{r}$  along the curve C in the xy – plane,  $y = x^3$  from point (1, 1) to (2, 8) (5 marks)
- (h) Show that the vector field  $\vec{F} = (2x + y)\hat{i} + x\hat{j} + 2z\hat{k}$  is conservative. (4 marks)

**Question Two (20 Marks)**

- a) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ,  $\vec{s} = x\hat{i} - y\hat{j} - 3z\hat{k}$  and  $\vec{t} = x\hat{i} + y\hat{j} + z\hat{k}$  (2 Marks)
- b) If  $\vec{r} = 2x\hat{i} - y\hat{j} + 3z\hat{k}$ , Compute  $\vec{\nabla} \cdot \vec{r} \cdot \vec{\nabla} \times \vec{r}$  at the point (1, 1, 1) (5 Marks)
- c) Let  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  and  $|\vec{r}| = r$ , then deduce the form taken by  $\vec{\nabla}$  (7 marks)
- d) Prove that  $\vec{\nabla} \cdot \vec{r} = 3$  (3 marks)

**Question Three (20 Marks)**

- a) Show that the vector field  $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$  is a gradient field. Hence or otherwise find the corresponding scalar potential. (10 Marks)
- b) Find the unit normal vector to  $\phi = x^2 + y^2 + z^2 = 1$  at (1, 1, 1) (5 Marks)

c) Calculate  $\text{div } 2\hat{x} - \hat{y} + 3\hat{z}$  (5 Marks)

**Question Four (20 Marks)**

a) Evaluate  $\int_C (x^2 + y^2) dz + (y - z) dx$  along the path  $x = t, y = t, z = t$  from  $t = 0$  to  $t = 1$ . (6 Marks)

b) If  $\vec{r} = (x^2 + y^2)\hat{x} + (z^2 + 2)\hat{y}$ , evaluate  $\int_C \vec{r} \cdot d\vec{r}$  along the curve  $x^2 + y^2 + z^2 = 1$  from the point  $(0, 1, 1)$  to  $(1, 0, 1)$  (9 Marks)

c) Evaluate  $\int_C z^2 dz + 2xy dx + 2yz dy$  where C is the segment of the line  $x^2 + y^2 = 2$  in the  $xy$ -plane from  $(-1, -2, 0)$  to  $(1, 2, 0)$  (5 Marks)

**Question Five (20 Marks)**

a) Use Green's theorem to evaluate  $\oint_C (3x + 4y) dx + (2x - 3y) dy$  where C is a circle of radius 2 with centre  $(0, 0)$  (9 Marks)

b) Evaluate  $\iint_S \vec{r} \cdot d\vec{S}$  where  $\vec{r} = 2x\hat{x} + y\hat{y} + z\hat{z}$  and S is the surface bounded by  $x = 0, y = 0, z = 0$  and  $x^2 + y^2 = 3$  (6 Marks)

c) Evaluate  $\iint_S \vec{r} \cdot d\vec{S}$  where  $\vec{r} = x\hat{x} + 2y\hat{y} - 4z\hat{z}$  and S is the surface of the plane  $2x + y = 6$  in the  $xy$ -plane and  $z = 4$  (5 Marks)