KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE AND BACHELOR OF SCIENCE ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 111

COURSE TITLE: VECTOR & GEOMETRY

STREAM: Y1S1

DAY: TUESDAY

TIME: 2.00 – 4.00 P.M.

DATE: 08/12/2009

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE

(a) Distinguish between as used in vectors

(i) Position and base vectors

(ii) Dot product and cross product **(4 marks)**

(b) Find magnitude and direction of the vectors

(i) $3\hat{\imath} + 4\hat{\jmath}$

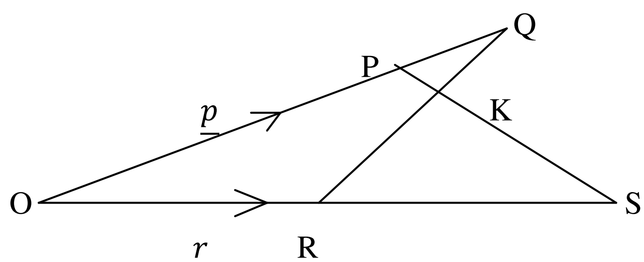
(ii) $-5\hat{\imath} + 12\hat{\jmath}$ **(4 marks)**

(c) In the figure below $\overrightarrow{0S} = 2\underline{r}$ and $\overrightarrow{OQ} = {}^{3}/_{2}\,\underline{p}$. Given that $\overrightarrow{QK} = m\overrightarrow{QR}$ and $\overrightarrow{PK} = m\overrightarrow{PS}$, find two

(i) Two distinct expression for $\overrightarrow{OK}$ **(4 marks)**

(ii) Find the values of $m\ and\ n$ **(3 marks)**

(iii) The ratios QR:KR and PK:KS **(2 marks)**



(d) Show that the line joining the midpoint of two sides of a triangle is parallel to the third side and half its length. **(3 marks)**

(e) Find the direction cosines $[l, m, n]$ of the vector $\underline{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{K}$ **(3 marks)**

(f) Given that A$(1, 1, 1)$ and B$(13, 4, 5)$ find;

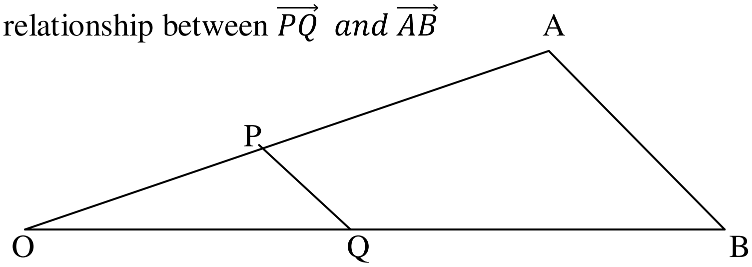
(i) The displacement vector $\overrightarrow{AB}$ in terms of $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$ **(2 marks)**

(ii) The unit vector parallel to $\overrightarrow{AB}$ **(2 marks)**

(g) Find the equation of the line through the points A$(1, 2, 3)$ and B$(4, 4, 4)$ **(3 marks)**

QUESTION TWO

(a) In triangle OAB below $\overrightarrow{OA} = \underline{a}$ *and* $\overrightarrow{OB} = \underline{b}$. Given that P and Q are the midpoints of OA and OB express $\overrightarrow{PQ}$ *and* $\overrightarrow{AB}$ in terms of $\underline{a}$ *and* $\underline{b}$ hence the geometrical relationship between $\overrightarrow{PQ}$ *and* $\overrightarrow{AB}$



(b) A stationary observer O observes a ship S at noon, at a point whose coordinate relative to O are$(20, 15)$; the units are kilometers. The ship is moving at as steady speed 10 km/h on a bearing 150°,

(i) Express its velocity as a column vector **(1 mark)**

(ii) Write down in terms of A, its position after t hours **(2 marks)**

(iii) Find the value of t when it is due East of O **(3 marks)**

(iv) How far is it at from O at this instants **(2 marks)**

(c) Show that the equations

$$\begin{pmatrix} y \\ -y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + m\begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix} \quad and$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \\ -3 \end{pmatrix} + n\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$$

represent the same line **(6 marks)**

(d) Find the centroid of the triangle whose vertices are

A$(1, 2, 3)$, $B(3, 7, 4)$ *and* $C(2, 0, 5)$

QUESTION THREE

(a) By defining the necessary relationships, show that the ration theorem is given by

$$\underline{c} = \frac{n}{m+n}\,\underline{a} + \frac{m}{m+n}\,\underline{b}$$ **(4 marks)**

(b) Given the equation of the line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-7}{12}$ find the unit vector parallel to it. **(4 marks)**

(c) Given four point A,B, C and D, the point G whose position vector g is given as

$$\underline{g} = {}^{1}\!/_{4}\,(\underline{a} + \underline{b} + \underline{c} + \underline{d})$$

(i) Show that G lies on the line joining D to M the centroid of triangle ABC **(3 marks)**

(ii) Find the ratio DG:GM **(3 marks)**

(d) Find the co-ordinates of the points where the line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$ meets the plane $x - 2y + 3z = 26$ **(6 marks)**

QUESTION FOUR

(a) Given that $\underline{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ show that the direction cosines [l, m, n] are given by

$$l = \frac{a}{r} \qquad m = \frac{b}{r} \qquad n = \frac{c}{r}$$ **(4 marks)**

(b) Given that $\underline{a} = 4\hat{\imath} + 3\hat{\jmath} + 12\hat{k}$ and $\underline{b} = 8\hat{\imath} - 6\hat{\jmath}$ find

(i) a^2, b^2 **(2 marks)**

(ii) $\underline{a} \cdot \underline{b}$ **(2 marks)**

(iii) The angle between the vectors $\underline{a}$ and $\underline{b}$ **(2 marks)**

(c) A destroyer sights a ship travelling with a constant velocity $5\hat{\imath}$, whose position vector at the time of sighting is $2000(3\hat{\imath} + \hat{\jmath})$ relative to the destroyer, distances being in m and velocity in ms^{-1}. The destroyer immediately begins to move with a velocity $k(4\hat{\imath} + 3\hat{\jmath})$, where k is a constant in order to intercept the ship.

(i) Find k and the time of interception **(3 marks)**

(ii) Find the distance between the vessels when half the time to intercept elapsed. **(3 marks)**

(d) If $\underline{a} = 5\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ and $\underline{b} = 4\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$ Find $\underline{a} \times \underline{b}$ **(- marks)**

QUESTION FIVE

(a) Show that $\underline{a} \cdot \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ **(4 marks)**

(b) Find the angle between the vectors $\underline{p} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\underline{q} = 4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ **(6 marks)**

(c) Find vertices of A B and C of a triangle have position vectors $\underline{a}$, $\underline{b}$, & $\underline{c}$ respectively relative to an origin O. The point P is on BC such that BP: PC = 3:1, the point Q is on CA such that CQ: QA = 2:3; the point R is on BA produced such that BR: AR = 2:1. The position vectors of PQ & R are $\underline{p}, \underline{q}$ & $\underline{r}$ respectively.

(i) Express $\underline{q}$ in terms of $\underline{p}$ and $\underline{r}$ **(2 marks)**

(ii) Show that P, Q & R are $\underline{p}, \underline{q}$ & $\underline{r}$ respectively. **(2 marks)**

(iii) State the ratio of the lengths PQ:QR **(2 marks)**

(e) Find the cosine of the angle between the vectors $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ & $3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$ **(4 marks)**