

KABARAK



UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF COMPUTER
SCIENCE**

COURSE CODE: MATH 111

COURSE TITLE: VECTOR GEOMETRY

STREAM: Y1S1

DAY: TUESDAY

TIME: 9.00 – 11.00 A.M.

DATE: 24/03/2009

INSTRUCTIONS:

Attempt **QUESTION ONE** and **ANY OTHER TWO** questions

PLEASE TURN OVER

Question one (30 Marks)

- a) Define the following terms: (3 Marks)
- (i) Unit vector.
 - (ii) Collinear vectors.
 - (iii) Coplanar vectors.
- b) $(2, -1, 3)$, $(8, 5, -6)$ and $R(4, 1, 0)$ are the vertices of a triangle. Show that $\vec{r} = 3\vec{s}$ and the direction cosines of \vec{r} are $\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, -\frac{2}{\sqrt{14}}$. (5 Marks)
- c) Find the unit vector perpendicular to the plane containing the vectors $\vec{r} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{s} = -3\hat{i} + 4\hat{j} + \hat{k}$. (4 Marks)
- d) Given $\vec{r} = 3\hat{i} - 4\hat{j} - 5\hat{k}$ and $\vec{s} = 2\hat{i} - 3\hat{j} + \hat{k}$ find:
- (i) $2\vec{r} - 3\vec{s}$. (1 Mark)
 - (ii) the projection of \vec{r} along \vec{s} . (3 Marks)
- e) Prove that $3\hat{i} - 2\hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} + 5\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$ form a right angled triangle. (4 Marks)
- f) Find the total work done when two forces $\vec{F}_1 = 2\hat{i} + 3\hat{j}$ and $\vec{F}_2 = 6\hat{i} + 7\hat{j} + 4\hat{k}$ displaces an object from point $(1, 4, 6)$ to $(3, 8, 5)$. (4 Marks)
- g) Show that:
- (i) $(\vec{r} - \vec{s}) \cdot (\vec{r} + \vec{s}) = \vec{r} \cdot \vec{r} - \vec{s} \cdot \vec{s}$.
 - (ii) $(\vec{r} - \vec{s}) \times (\vec{r} + \vec{s}) = 2\vec{r} \times \vec{s}$. (6 Marks)

Question Two (20 Marks)

- a) Show that the quadrilateral with vertices at $(5, 2, 0)$, $(2, 6, 1)$, $(2, 4, 7)$, and $D(5, 0, 6)$ is a parallelogram and then find its area. Is it a rectangle? (6 Marks)
- b) Find the angle between the planes given by $2x = -y + 4z$ and $3x = 2y + 2z + 8$. (4 Marks)
- c) Determine whether the three vectors $\vec{r} = (1, 4, -7)$, $\vec{s} = (2, -1, 4)$, and $\vec{t} = (0, -9, 18)$ lie on the same plane or not. (6 Marks)
- d) Show that $\vec{r} \times \vec{s} \times \vec{t} + \vec{s} \times \vec{t} \times \vec{r} + \vec{t} \times \vec{r} \times \vec{s} = 0$. (4 Marks)

Question Three (20 Marks)

- a) Find the angle between the lines $\vec{r} = 4\hat{i} - \hat{j} + \hat{k} + 2\hat{i} - 2\hat{j}$ and $\vec{r} = \hat{i} - \hat{j} + 2\hat{k} - 2\hat{i} + 4\hat{j} + 4\hat{k}$. (4 Marks)
- b) A plane moves Northward at 40 km/h. There is a wind that is blowing at 15 km/h 30° east of North. Determine:
- (i) The resultant velocity of the plane.
 - (ii) The direction of the plane. (7 Marks)

Question Four (20 Marks)

- a) Find the equation of a plane that contains the points (3,4,2), (6,2,0) and (1,4,1). (6 Marks)
- b) Prove that $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$. (4 Marks)
- c) Find the area of the triangle having vertices P(1,3,2), Q(2,-1,1), and R(-1,2,3). (5 Marks)
- d) State and prove the triangle inequality. (5 Marks)

Question Five (20 Marks)

- a) Using vectors prove the cosine rule. (5 Marks)
- b) Write the equation of the line that passes through the points (2,-1,3) and (1,4,-3) in:
- (i) the vector form.
 - (ii) the parametric form.
 - (iii) the symmetric form.
 - (iv) Where does the line intersect the xy-plane? (7 Marks)
- c) Given two vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ derive their dot product. (3 Marks)
- d) Find the intersection of the line with the parametric equations $x = 2 + 3t$, $y = -3 + 5t$, $z = 4 - 6t$ and the plane $2x - 3y - 3z = 4$. (5 Marks)