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## 2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF ECONOMICS \& MATHEMATICS
COURSE CODE: MATH 111
COURSE TITLE: VECTOR GEOMETRY
STREAM: ..... Y1S1
DAY: MONDAY
TIME: 11.00 - 1.00 P.M.
DATE: 15/12/2008
INSTRUCTIONS TO CANDIDATES:

1. Answer Question ONE and any other TWO Questions

## QUESTION ONE (30 MARKS)

(a) Distinguish between coplanar vectors and resultant vectors.
(4 mks)
(b) Using vectors show that

$$
\operatorname{Sin}(A-B)=\operatorname{Sin} A \operatorname{Cos} B-\operatorname{Cos} A \operatorname{Sin} B
$$

(c) Find the angle between the plane

$$
\begin{align*}
& 2 x+3 y+6 z=12 \text { and the line } \\
& x=4+2 t, y=3+2 t \text { and } z=2-t \tag{4mks}
\end{align*}
$$

(d) Show that;

$$
\begin{equation*}
x \rightarrow X|\vec{A} X \vec{B}|^{2}+|\vec{A} \cdot \vec{B}|^{2}=|\vec{A}|^{2}|\vec{B}|^{2} \tag{4mks}
\end{equation*}
$$

(e) Find the work done by forces $\vec{F}_{1}=\overrightarrow{2 i}+\overrightarrow{3 j}-\overrightarrow{6 k}$ and $\vec{F}_{2}=\vec{i}+\overrightarrow{4 j}-\vec{k}$ in displacing an object from $(1,5,4)$ to point $(3,8,9)$
(f) A triangle is defined by the point $\mathrm{A}(0,1,0), \mathrm{B}(1,0,1)$ and $\mathrm{C}(0,1,4)$

Find the area of the triangle and a unit a vector perpendicular to the plane containing the triangle.
(g) Find the angle between the planes $x+2 y+6 z=8$ and $2 x+4 y+z=-4$

## QUESTION TWO (20 MARKS)

(a) Show that the diagonals of a parallelogram bisect each other.
(b) Prove that $[(\vec{a}+\vec{b}+\vec{c}) X(\vec{b}+\vec{c}) \cdot \vec{c}=\vec{a} \cdot(\vec{b} \times \vec{c})]$
(c) Find a unit vector perpendicular to the plane containing the points $(1,4,1)(0,6,3)$ and $(3,2,4)$ using dot product method
(d) Determine whether the points $\mathrm{A}(0,2,5) \mathrm{B}(1,6,3)$ and $\mathrm{C}(4,8,1)$ are collinear.
( 2 mks )

## QUESTION THREE (20 MKS)

(a) Prove that the medians of a triangle bisect each other in the ration 2:1
(b) Determine the projection of

$$
\vec{b}=\overrightarrow{2 j}+\overrightarrow{4 j}, \vec{k} \text { onto } \vec{a}+\vec{i}-2 \vec{k}
$$

Interpret the result.
(c) Find the distance between the point $(0,4,6)$ and the line passing through $(3,4,6)$ and $(0,3,2)$

## QUESTION FOUR (20 MKS)

(a) Derive the formula for determining the volume of a parallelepiped.
(b) A parallelogram has vertices $(0,0,0),(6,1,1),(8,5,2)$ and $(2, y, x)$
(i) Find the value of $x$ and $y$
(ii) Find the area of the parallelogram
(c) Derive the formula for determining the position vector of the centroid of a triangle. Use it to find the position vector of the centroid of a triangle with points $\mathrm{A}(2,4,7)$ $\mathrm{B}(3,-4,0)$ and $\mathrm{C}(4,0,2)$.
(d) Prove the law of cosines.

## QUESTION FIVE (20 MARKS)

(a) Find the equation of a plane that contains the points $\mathrm{A}(2,5,1), \mathrm{B}(0,2,1)$ and $\mathrm{C}(3,6,2)$
(b) Find a vector that is parallel to the line of intersection.

$$
\begin{aligned}
& 3 x-6 y-2 z=15 \\
& 2 x+y-2 z=5
\end{aligned}
$$

Hence find the parametric equation of the line parallel to the line of intersection.
(c) Calculate

$$
\begin{equation*}
(\overrightarrow{2 i}+3 \vec{j}+\vec{k}) \times(\overrightarrow{2 i}+\vec{j}) \times(\vec{j}+\overrightarrow{3 k}) \tag{3mks}
\end{equation*}
$$

(d) Determine whether the vectors are coplanar

$$
\begin{array}{ll}
\vec{a}=\overrightarrow{2 i}+3 \vec{j}+\overrightarrow{4 k} & \vec{r}=\overrightarrow{7 i}-\overrightarrow{6 j}+1 \overrightarrow{4 k} \\
\vec{b}=\vec{i}-\overrightarrow{4} j+2 \vec{k} & \tag{4mks}
\end{array}
$$

