KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE AND BACHELOR OF ECONOMIC & MATHEMATICS

COURSE CODE: **MATH 111**

COURSE TITLE: **VECTOR GEOMETRY**

STREAM: **Y1S1**

DAY: **MONDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **22/03/2010**

INSTRUCTIONS:

1. Question ONE is compulsory.
2. Attempt question ONE and any other TWO

PLEASE TURN OVER

Question One [30 Marks]

a) Given that $\underline{r_1} = 2i - \hat{j} + \hat{k}$, $\underline{r_2} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\underline{r_3} = -2\hat{i} + \hat{j} - 3\hat{k}$ and $\underline{r_4} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ find the scalars *a, b, c* such that $\underline{r_4} = a\underline{r_1} + b\underline{r_2} + c\underline{r_3}$ [4 marks]

b) Find the sum or resultant of the following displacement
A; 10 m northwest, B; 20 m 30^0 north of east and C; 35 m due south. [3 Marks]

c) Given that A is the point (1, 3) and that $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ respectively, find the coordinates of the vertices B, C and D of the parallelogram ABCD.
[3 Marks]

d) Find the centroid of the triangle whose vertices are $A(1,2,3)$, $B(3,7,4)$ and $C(2,0,5)$
[3 Marks]

e) Find the equation of through the points $P(2,4,5)$ and $Q(7,9,4)$ hence find the coordinates of the point where this line meets the plane z = 0. [5 marks]

f) Determine the value of a so that the vectors $\underline{p} = 2\hat{i} + a\hat{j} + 4\hat{k}$ and $q = 5\hat{i} + 2\hat{j} - 4\hat{k}$ are perpendicular. [2 Marks]

g) Find the vector product of $\underline{p}$ and $\underline{q}$ where $\underline{p} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\underline{q} = 2\hat{i} + 5\hat{j} - \hat{k}$
[6 Marks]

h) Use vectors to prove the law of sines for plane triangles. [4 Marks]

Question Two [20 Marks]

a) An automobile travels 3 km due north then 5 km northeast. Represent these displacements graphically and hence or otherwise determine the resultant displacement. [4 marks]

In a triangle OAB, X is a point on OB such that $\overrightarrow{OX} = 2\overrightarrow{XB}$ and Y is a point AB such that $2\overrightarrow{BY} = 3\overrightarrow{YA}$.

i. Express **x** and **y** in terms of **a** and **b**. [2 marks]

ii. Find the position vector of any point on XY and hence find the position vector of the point Z, where XY produced meets OA produced. Calculate the value of AZ/OZ [4 marks]

b) AT noon two boats P and Q are at points where position vectors are $4\hat{i}+8\hat{j}$ and $4\hat{i}+3\hat{j}$ respectively. Both boats are moving with a constant velocity; the velocity of P is $4\hat{i}+\hat{j}$ and the velocity of Q is $2\hat{i}+5\hat{j}$ where all distances are in kilometers and time measured in hours.

i) Find the position vectors of P and Q and $\overrightarrow{PQ}$ after *t* hours. [4 Marks]

ii) Express the distance PQ between the boats in terms of *t* [2 Marks]

iii) Show that the least distance between the boats is $\sqrt{5}$ km [4 Marks]

Question Three [20 Marks]

a) Show that addition of vectors is commutative. [2 marks]

b) If $\underline{c}=\frac{2}{5}\underline{a}+\frac{3}{5}\underline{b}$, show that C is a point on AB and that AC:CB = 3:2 [2 Marks]

c) Given that A and B are the points (1, 1, 1) and (13, 4, 5) respectively,

i. Find in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$ the displacement vector $\overrightarrow{AB}$ [2 Marks]

ii. Find the unit vector parallel to $\overrightarrow{AB}$ [4 Marks]

iii. Find the point P on $\overrightarrow{AB}$ such that AP:PB = 1:3 [4 Marks]

d) Two forces $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ of magnitudes 5.0 kilograms and 8.0 kilograms respectively act at a point P. The direction of $\overrightarrow{PQ}$ is N20^0E, and the direction of $\overrightarrow{PR}$ is N65^0E. Approximate the magnitude and direction of the resultant $\overrightarrow{PS}$ [6 marks]

Question Four [20 Marks]

a) Determine the angles $\alpha,\beta,and\gamma$ which the vector $\underline{r}=x\hat{i}+y\hat{j}+z\hat{k}$ makes with the positive directions of the coordinate axis and hence show that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$ [5 Marks]

b) Show that $\underline{a}\bullet\underline{b}=a_1b_1+a_2b_2+a_3b_3$ given that $\underline{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\underline{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ [4 Marks]

c) Given that A, B and C are the points (1, 1, 1), (5, 0, 0) and (3, 2, 1) respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point P in the plane ABC. [6 Marks]

d) Find the equation of the line of intersection given that the equation of two non-parallel planes as $2x - 3y + z = 3$ and $3x - 5y + z = 8$ [5 Marks]

Question Five [20 Marks]

a) Show that $A \times B \times C = B(A \bullet C) - C(A \bullet B)$ [5 Marks]

b) Given that $\underline{a} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ and $\underline{b} = 8\hat{i} - 6\hat{j}$ find

i. $\underline{a} \bullet \underline{b}$ [2 Marks]

ii. The angle between the two vectors $\underline{a}$ and $\underline{b}$ [3 Marks]

c) Find an equation for the plane perpendicular to the vector $\underline{a} = 2\hat{i} + 3\hat{j} + 16\hat{k}$ and passing through the terminal point of the vector $\underline{b} = \hat{i} + 5\hat{j} + 13\hat{k}$. Hence find the distance from the origin to the plane. [6 marks]

d) Determine a unit vector perpendicular to the plane of $\underline{p} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\underline{q} = 4\hat{i} + 3\hat{j} - \hat{k}$ [4 marks]