KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: MATH 111

COURSE TITLE: VECTORS & GEOMETRY

STREAM: Y1S1

DAY: WEDNESDAY

TIME: 2.00 – 4.00 P.M.

DATE: 23/03/2011

INSTRUCTIONS:

1. Question **ONE** is compulsory.
2. Attempt question **ONE** and any other **TWO** Questions

PLEASE TURNOVER

QUESTION ONE [30 MARKS]

i. Given that $\underline{r_1} = 2i - \hat{j} + \hat{k}$, $\underline{r_2} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\underline{r_3} = -2\hat{i} + \hat{j} - 3\hat{k}$ and $\underline{r_4} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ find the magnitude of $2\underline{r_1} - 3\underline{r_2} - 5\underline{r_3}$ [4 marks]

ii. Show that the magnitude of the vector $A = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ is given by $\left|\bar{A}\right| = \sqrt{A_1^2 + A_2^2 + A_3^2}$

[3 Marks]

iii. Given that the wind is blowing at 12 miles/ hour in the direction N40^0W, express its velocity as a vector. [3 Marks]

iv. Find the work done in moving an object along a vector $\underline{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ if the applied force is $\overrightarrow{F} = 2\hat{i} - \hat{j} - \hat{k}$ [3 Marks]

v. Determine a unit vector that is perpendicular to the plane of $A = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $B = 4\hat{i} + 3\hat{j} - \hat{k}$. [5 marks]

vi. Find the value of a such that the pair of vectors are orthogonal $\underline{p} = 2\hat{i} + a\hat{j} + 4\hat{k}$ and $\underline{q} = 5\hat{i} + 2\hat{j} - 4\hat{k}$ [2 Marks]

vii. Find the direction cosines of the resultant vector of $\underline{p} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\underline{q} = 2\hat{i} + 5\hat{j} - \hat{k}$

[3 Marks]

viii. The centroid of triangle OAB is denoted by G. If O is the origin and $\overline{OA} = 4\hat{i} + 3\hat{j}$ $\overline{OB} = 6\hat{i} - \hat{j}$ find $\overline{OG}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$ [3 Marks]

ix. Find the angle between the vectors $\underline{a} = 2\hat{i} + 3\hat{j}$ and $\underline{b} = 5\hat{i} + \hat{j}$ [4 Marks]

QUESTION TWO [20 MARKS]

a) Evaluate $(2\hat{i} - 3\hat{j}) \bullet \left(\hat{i} + \hat{j} - \hat{k}\right) \times \left(3\hat{i} - \hat{k}\right)$. [4 Marks]

b) An automobile travels 3 km due north then 5 km northeast. Represent these displacements graphically and hence or otherwise determine the resultant displacement. [4 marks]

c) A stationery observer O observes a ship S at noon at a point whose coordinates relative to O are $(20,15)$; units are in kilometers. The ship is moving at a steady speed of 10 km/h on a bearing 150^0.

i) Express its velocity as a column vector. [3 Marks]

ii) Write down in terms of *t*, its position after *t* hours [3 Marks]

iii) Find the value of t when the ship is due East [3 Marks]

iv) How far is it from O at this instant [3 Marks]

QUESTION THREE [20 MARKS]

a) Points L, M, N are the mid-points of the sides AB, BC, CA of the triangle ABC. Show that

$2\overrightarrow{AB} + 3\overrightarrow{BC} + \overrightarrow{CA} = 2\overrightarrow{LC}$ [3 Marks]

b) Given the equation of the line in the form $\frac{x-3}{5} = \frac{y-2}{3} = \frac{z+2}{7} = t$

i. Express the equation in the form $\underline{r} = \underline{a} + t\underline{u}$ [3 Marks]

ii. Show that the line passes through $(8,14,11)$ [3 Marks]

c) In a triangle OAB, X is a point on OB such that OX = 2XB and Y is a point on AB such that 2BY = 3 YA.

i. Express x and y in terms of a and b [2 Marks]

ii. Find the position vector of any point on XY [3 Marks]

iii. Find the position vector of the point Z, where XY produced meets OA produced. [3 Marks]

iv. Calculate the value of AZ/OZ [3 Marks]

QUESTION FOUR [20 MARKS]

a) Show that $\underline{a} \bullet \underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ given that $\underline{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\underline{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

[4 Marks]

b) Given that $\underline{a} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ and $\underline{b} = 8\hat{i} - 6\hat{j}$ find

i. $\underline{a} \bullet \underline{b}$ [2 Marks]

ii. The angle between the two vectors $\underline{a}$ and $\underline{b}$ [3 Marks]

c) Given that A, B and C are the points (1, 1, 1), (5, 0, 0) and (3, 2, 1) respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point P in the plane ABC.

[6 Marks]

d) Find the equation of the line of intersection given that the equation of two non-parallel planes as $2x-3y+z=3$ and $3x-5y+z=8$ [5 Marks]

QUESTION FIVE [20 MARKS]

a) Show that $\left|\breve{A}\times\breve{B}\right|^2+\left|\breve{A}\bullet\breve{B}\right|^2=\left|\breve{A}\right|^2\left|\breve{B}\right|^2$ [5 Marks]

b) Find an equation for the plane perpendicular to the vector $\underline{a}=2\hat{i}+3\hat{j}+16\hat{k}$ and passing through the terminal point of the vector $\underline{b}=\hat{i}+5\hat{j}+13\hat{k}$. Hence find the distance from the origin to the plane. [6 marks]

c) Find the area of a triangle having vertices $P(1,3,2)$ $Q(2,-1,1)$ $R(-1,2,3)$ [4 marks]

d) Find the volume of a parallelepiped whose edges are represented by $A=2\hat{i}-3\hat{j}+4\hat{k}$

$B=\hat{i}+2\hat{j}-\hat{k}$ and $C=2\hat{i}-\hat{j}+2\hat{k}$ [5 Marks]