

KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF COMPUTER
SCIENCE**

COURSE CODE: MATH 111

COURSE TITLE: VECTORS & GEOMETRY

STREAM: Y1S1

DAY: MONDAY

TIME: 2.00 – 4.00 P.M.

DATE: 02/08/2010

INSTRUCTIONS:

1. Question ONE is compulsory.
2. Attempt question ONE and any other TWO Questions

PLEASE TURNOVER

Question One [30 Marks]

- a) Given that $\underline{r}_1 = 2\hat{i} - \hat{j} + \hat{k}$, $\underline{r}_2 = \hat{i} + 3\hat{j} - 2\hat{k}$, $\underline{r}_3 = -2\hat{i} + \hat{j} - 3\hat{k}$ and $\underline{r}_4 = 3\hat{i} + 2\hat{j} + 5\hat{k}$ find the scalars a, b, c such that $\underline{r}_4 = a\underline{r}_1 + b\underline{r}_2 + c\underline{r}_3$ [4 marks]
- b) Find the sum or resultant of the following three displacement
A; 10 m northwest, B; 20 m 30° north of east and C; 35 m due south. [3 Marks]
- c) Given that the wind is blowing at 12 miles/ hour in the direction $N40^\circ W$, express its velocity as a vector. [3 Marks]
- d) Find a vector \underline{b} in the opposite direction of $\underline{a} = \langle 5, -12 \rangle$ that has magnitude 6 [3 Marks]
- e) Find the equation of the line through the points $P(2,4,5)$ and $Q(7,9,4)$ hence find the coordinates of the point where this line meets the plane $z = 0$. [5 marks]
- f) Show that the following pair of vectors are orthogonal $\underline{p} = 2\hat{i} + a\hat{j} + 4\hat{k}$ and $q = 5\hat{i} + 2\hat{j} - 4\hat{k}$ [2 Marks]
- g) Find the vector product of \underline{p} and \underline{q} where $\underline{p} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\underline{q} = 2\hat{i} + 5\hat{j} - \hat{k}$ [6 Marks]
- h) Find the angle between the vectors $\underline{a} = 2\hat{i} + 3\hat{j}$ and $\underline{b} = 5\hat{i} + \hat{j}$ [4 Marks]

Question Two [20 Marks]

- a) Prove the law of sines for triangles. [4 marks]
- b) An automobile travels 3 km due north then 5 km northeast. Represent these displacements graphically and hence or otherwise determine the resultant displacement. [4 marks]
- c) AT noon two boats P and Q are at points where position vectors are $4\hat{i} + 8\hat{j}$ and $4\hat{i} + 3\hat{j}$ respectively. Both boats are moving with a constant velocity; the velocity of P is $4\hat{i} + \hat{j}$ and the velocity of Q is $2\hat{i} + 5\hat{j}$ where all distances are in kilometers and time measured in hours.
- i) Find the position vectors of P and Q and \overrightarrow{PQ} after t hours. [4 Marks]
- ii) Express the distance PQ between the boats in terms of t [2 Marks]
- iii) Show that the least distance between the boats is $\sqrt{5}$ km [4 Marks]

Question Three [20 Marks]

- a) Given the midpoints of the consecutive sides of any quadrilateral are connected by straight lines prove that the resulting quadrilateral is a parallelogram [2 Marks]
- b) Given that A and B are the points (1, 1, 1) and (13, 4, 5) respectively,
- Find in terms of \hat{i} , \hat{j} and \hat{k} the displacement vector \overrightarrow{AB} [2 Marks]
 - Find the unit vector parallel to \overrightarrow{AB} [4 Marks]
 - Find the point P on \overrightarrow{AB} such that AP:PB = 1:3 [4 Marks]
- c) A small cart weighing 100 kg is pushed up an incline that makes an angle of 30° with the horizontal. Find the work done against the gravity in pushing the cart a distance of 80 metres [6 marks]

Question Four [20 Marks]

- a) Determine the angles $\alpha, \beta,$ and γ which the vector $\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$ makes with the positive directions of the coordinate axis and hence show that
- $$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \quad [5 \text{ Marks}]$$
- b) Show that $\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ given that $\underline{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\underline{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ [4 Marks]
- c) Given that A, B and C are the points (1, 1, 1), (5, 0, 0) and (3, 2, 1) respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point P in the plane ABC. [6 Marks]
- d) Find the equation of the line of intersection given that the equation of two non-parallel planes as $2x - 3y + z = 3$ and $3x - 5y + z = 8$ [5 Marks]

Question Five [20 Marks]

- a) Show that $\underline{A} \times \underline{B} \times \underline{C} = \underline{B}(\underline{A} \cdot \underline{C}) - \underline{C}(\underline{A} \cdot \underline{B})$ [5 Marks]
- b) Given that $\underline{a} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ and $\underline{b} = 8\hat{i} - 6\hat{j}$ find
- $\underline{a} \cdot \underline{b}$ [2 Marks]
 - The angle between the two vectors \underline{a} and \underline{b} [3 Marks]

c) Find an equation for the plane perpendicular to the vector $\underline{a} = 2\hat{i} + 3\hat{j} + 16\hat{k}$ and passing through the terminal point of the vector $\underline{b} = \hat{i} + 5\hat{j} + 13\hat{k}$. Hence find the distance from the origin to the plane. [6 marks]

d) Determine a unit vector perpendicular to the plane of $\underline{p} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\underline{q} = 4\hat{i} + 3\hat{j} - \hat{k}$ [4 marks]