KABARAK   **UNIVERSITY**

UNIVERSITY EXAMINATIONS
2010/2011 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF
COMPUTER SCIENCE

COURSE CODE: MATH 111

COURSE TITLE: VECTORS & GEOMETRY

STREAM: Y1S1

DAY: **TUESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **07/12/2010**

INSTRUCTIONS:

1.Question **ONE** is compulsory.
2. Attempt question **ONE** and any other **TWO** Questions

PLEASE TURNOVER

Question One   [30 Marks]

a) Given that $r_1 = 2i - \hat{j} + \hat{k}$, $r_2 = \hat{i} + 3\hat{j} - 2\hat{k}$, $r_3 = -2\hat{i} + \hat{j} - 3\hat{k}$ and $r_4 = 3\hat{i} + 2\hat{j} + 5\hat{k}$

   find the magnitude of $2r_1 - 3r_2 - 5r_3$ **[4 marks]**

b) Show that the magnitude of the vector $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ is given by

   $$\left|\vec{A}\right| = \sqrt{A_1^2 + A_2^2 + A_3^2}$$ **[3 Marks]**

c) Given that the wind is blowing at 12 miles/ hour in the direction N40^{0}W, express its velocity as a vector.   **[3 Marks]**

d) Find a vector b in the opposite direction of $a = \langle 5, -12 \rangle$ that has magnitude 6

   [3 Marks]

e) Determine a unit vector that is perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and

   $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$.   **[5 marks]**

f) Show that the pair of vectors are orthogonal $p = 2\hat{i} + a\hat{j} + 4\hat{k}$ and $q = 5\hat{i} + 2\hat{j} - 4\hat{k}$

   [2 Marks]

g) Find the vector product of p and q where $p = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $q = 2\hat{i} + 5\hat{j} - \hat{k}$

   [6 Marks]

h) Find the angle between the vectors $a = 2\hat{i} + 3\hat{j}$ and $b = 5\hat{i} + \hat{j}$ **[4 Marks]**

Question Two   [20 Marks]

a)   Prove the law of sines for triangles.   **[4 marks]**

b)   An automobile travels 3 km due north then 5 km northeast. Represent these displacements graphically and hence or otherwise determine the resultant displacement.   **[4 marks]**

c) AT noon two boats P and Q are at points where position vectors are $4\hat{i}+8\hat{j}$ and $4\hat{i}+3\hat{j}$ respectively. Both boats are moving with a constant velocity; the velocity of P is $4\hat{i}+\hat{j}$ and the velocity of Q is $2\hat{i}+5\hat{j}$ where all distances are in kilometers and time measured in hours.

   i)   Find the position vectors of P and Q and $\overrightarrow{PQ}$ after t hours. **[4 Marks]**

   ii)   Express the distance PQ between the boats in terms of t **[2 Marks]**

   iii)   Show that the least distance between the boats is $\sqrt{5}$ km   **[4 Marks]**

Question Three   [20 Marks]

a) Given the midpoints of the consecutives sides of any quadrilateral are connected by straight lines prove that the resulting quadrilateral is a parallelogram   **[2 Marks]**

b) Given that A and B are the points (1, 1, 1) and (13, 4, 5) respectively,

   i.   Find in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$ the displacement vector $\overrightarrow{AB}$ **[2 Marks]**

   ii.   Find the unit vector parallel to $\overrightarrow{AB}$ **[4 Marks]**

   iii. Find the point P on $\overrightarrow{AB}$ such that AP:PB = 1:3   **[4 Marks]**

c) A small cart weighing 100 kg is pushed up an incline that makes an angle of 30^0 with the horizontal. Find the work done against the gravity in pushing the cart a distance of 80 metres   **[6 marks]**

Question Four [20 Marks]

a) Determine the angles $\alpha, \beta, and \gamma$ which the vector $\underline{r}=x\hat{i}+y\hat{j}+z\hat{k}$ makes with the positive directions of the coordinate axis and hence show that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$ **[5 Marks]**

b) Show that $\underline{a}\bullet\underline{b}=a_1b_1+a_2b_2+a_3b_3$ given that $\underline{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and

$\underline{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ **[4 Marks]**

c) Given that A, B and C are the points (1, 1, 1), (5, 0, 0) and (3, 2, 1) respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point P in the plane ABC.   **[6 Marks]**

d) Find the equation of the line of intersection given that the equation of two non-parallel planes as $2x - 3y + z = 3$ and $3x - 5y + z = 8$ **[5 Marks]**

Question Five [20 Marks]

a) Show that $A \times B \times C = B(A \bullet C) - C(A \bullet B)$ **[5 Marks]**

b) Given that $\underline{a} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ and $\underline{b} = 8\hat{i} - 6\hat{j}$ find

   i. $\underline{a} \bullet \underline{b}$ **[2 Marks]**

   ii.   The angle between the two vectors $\underline{a}$ and $\underline{b}$ **[3 Marks]**

c) Find an equation for the plane perpendicular to the vector $\underline{a} = 2\hat{i} + 3\hat{j} + 16\hat{k}$ and passing through the terminal point of the vector $\underline{b} = \hat{i} + 5\hat{j} + 13\hat{k}$. Hence find the distance from the origin to the plane. **[6 marks]**

d) Determine a unit vector perpendicular to the plane of $\underline{p} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\underline{q} = 4\hat{i} + 3\hat{j} - \hat{k}$ **[4 marks]**