KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS & BACHELOR OF COMPUTER SCIENCE

COURSE CODE: MATH 211

COURSE TITLE: LINEAR ALGEBRA I

STREAM: Y2S1 & Y2S2

DAY: WEDNESDAY

TIME: 9.00 – 11.00 A.M.

DATE: 16/03/2011

INSTRUCTIONS:

- Answer question **ONE** and any other **TWO** questions
- Begin each question on a separate page
- Show your workings clearly

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

a) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Find i) A + B (2 marks)
ii) B + A (2 marks)
iii) (A + B) + C = A + (B + C) (3 marks)

b) Find the angle between the vectors

a = i + 2j + 3k and b = i – j + 2k (4 marks)

c) Solve for x from the following

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$ (5 marks)

d) Find the rank of the following matrix

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 \\ 1 & 3 & 4 & 2 \\ 2 & 5 & 1 & 3 \end{bmatrix}$$ (4 marks)

e) Show that u = (12,0,4,0,16,-4) and v = (-8,20,0,8,-12,-72) are orthogonal and verify that the pythagorean theorem holds (5 marks)

f) Given that u = (-6, 9, 3,-3) and v = (21, 3,-12,-6) verify the cauchy-schwarz inequality and the triangle inequality (5 marks)

QUESTION TWO (20 MARKS)

Find the inverse of the following matrix using row reduction method

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

QUESTION THREE (20 MARKS)

a) Suppose that x = (x_1, x_2 , x_3) and y = (y_1, y_2, y_3) are two vectors in three space show that

$x.y = x_1y_1 + x_2y_2 + x_3y_3$ (8 marks)

b) Find the determinat of the following

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$ (4 marks)

c) If a = 2i – j + 2k and b = 10i – 2j + 7k. Find a x b , b x a and also find the unit vector perpendicular to both. (8 marks)

QUESTION FOUR (20 MARKS)

a) Find the sine of the angle between the vectors 3i + j + 2k and 2i -2j +4k (7 marks)

b) If A = $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}$ and k_1 =i , k_2 =2

Verify $(k_1 + k_2) A = k_1A + k_2A$ (5 marks)

c) Given u = (5,2) , v = (0,7) and w = (4,10) compute

i) u.u and $\|u\|^2$ (3 marks)

ii) u.w (2 marks)

iii) (-2u).v and u.(-2v) (3 marks)

QUESTION FIVE (20 MARKS)

a) Suppose that u and v are two vectors in $\Re^n$ and that c is a scalar show that

i) $\|cu\| = |c|\|u\|$ (4 marks)

ii) $\|u + v\| \leq \|u\| + \|v\|$ (4 marks)

b)

$$2x_1 - 3x_2 + 2x_3 = 9$$

$$3x_1 + 2x_2 - x_3 = 4$$

$$x_1 - 4x_2 + 2x_3 = 6$$

Use Gaussian elimination and Gauss-Jordan elimination methods to solve the above system of equations (12 marks)