

KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF COMPUTER
SCIENCE AND BACHELOR OF SCIENCE IN
ECONOMICS & MATHEMATICS**

COURSE CODE: MATH 211

COURSE TITLE: LINEAR ALGEBRA I

STREAM: Y2S2 & Y2S1

DAY: THURSDAY

TIME: 9.00 – 11.00 A.M.

DATE: 12/08/2010

INSTRUCTIONS:

- Answer question ONE and any other TWO questions
- Begin each question on a separate page
- Show your workings clearly

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

a) Consider the matrices A and B $A = \begin{bmatrix} 2 & 4 \\ -6 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -5 \\ -3 & 2 \end{bmatrix}$

Find a) $A - B$ (4 marks) b) $\frac{1}{2}A + B$ (5 marks)

b) Let A be an invertible matrix. Show that $\det(A^{-1}) = \frac{1}{\det(A)}$ (4 marks)

c) Given $\mathbf{u}=(6,-2,8)$ and $\mathbf{v}=(4,0,2)$ compute each of the following

$\mathbf{u} \times \mathbf{v}$ and $\mathbf{v} \times \mathbf{u}$ (7 marks)

d) Find the angle between the vectors $\mathbf{p} = 4\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$ and $\mathbf{q} = 8\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$ (6 marks)

e) Show that (p_1, p_2, p_3) is a spanning set for p_2 , where $p_1(x) = 2 + 3x + x^2$, $p_2 = 4 - x$, $p_3 = -1$ (4 marks)

QUESTION TWO (20 MARKS)

Consider the linear system

$$x_1 - x_2 + 3x_3 = 2$$

$$2x_1 - x_2 = 5$$

$$-x_2 + 2x_3 - 6x_3 = 0$$

a) Write down the augmented matrix of the system. (2 marks)

b) Use Gaussian elimination to bring the augmented matrix to row echelon form and indicate which elementary row operations are used in each step. (9 marks)

c) Proceed to obtain the solution using reduced row echelon form using Gauss- Jordan elimination (9 marks)

QUESTION THREE (20 MARKS)

a) Use crammers rule to find the solution of the following

$$x - y - 3z = 2$$

$$x + y + 4z = 3$$

$$-x + 2y + 5z = -1$$
 (10 marks)

b) Determine which of the following collection of vectors are linearly independent

i) $(1,1,1), (3,4,3)^T, (2,1,3)^T, (1,1,3)^T$ (5 marks)

ii) $(2, -1, 5)^T, (1,3,2)^T, (3,2,7)^T$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Which of the following are spanning sets

i) $(1,0,0)^T, (0,0,1)^T, (1,2,4)^T$ (3 marks)

ii) $(1,1,1)^T, (1,1,0)^T, (1,0,0)^T$ (3 marks)

iii) $(1,2,4)^T, (2,1,3)^T, (4,-1,1)^T$ (4 marks)

b) Find the inverse of the following using determinant method

$$A = \begin{bmatrix} 1 & 3 & -1 \\ -2 & -6 & 0 \\ 1 & 4 & -3 \end{bmatrix} \quad (10 \text{ marks})$$

QUESTION FIVE (20 MARKS)

a) Consider the following vectors in \mathbb{R}^3

$$b_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad b_3 = \begin{bmatrix} -3 \\ 6 \\ 1 \end{bmatrix} \quad \text{and} \quad d_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \quad d_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad d_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Show that (b_1, b_2, b_3) and (d_1, d_2, d_3) are bases of \mathbb{R}^3 (6 marks)

b) Determine the angle between the following vectors

i) $\mathbf{a} = (18, -4)$ and $\mathbf{b} = (8, 36)$ (5 marks)

ii) $\mathbf{u} = (6, -2, 12)$ $\mathbf{v} = (8, 4, 0)$ (5 marks)

a) c) Compute the norms of the given vectors

i) $\mathbf{v} = (-10, 6, 18)$ (2 marks)

ii) $\mathbf{j} = (0, 2, 0)$ (2 marks)