## DAY:

TIME:
9.00 - 11.00 A.M.

DATE:
24/03/2010

## INSTRUCTIONS:

1. Answer question ONE and any other TWO questions
2. Begin each question on a separate page
3. Show your workings clearly and neatly.

## QUESTION ONE (30 MARKS)

a) Find the determinant of $\mathrm{A}=\left(\begin{array}{lll}2 & -1 & 1 \\ -1 & 0 & 3 \\ 2 & 1 & -4\end{array}\right)$
b) Compute the inverse of the following matrices using row reduction method
i) $\quad\left(\begin{array}{l}3 \\ 5\end{array}\right.$
$\left.\begin{array}{l}1 \\ 2\end{array}\right)$ (3 marks)
ii) $\left(\begin{array}{l}2 \\ 4\end{array}\right.$
$\left.\begin{array}{l}-3 \\ 4\end{array}\right)$
c) Given $\mathbf{v}=(3,-1,-2)$ find the a unit vector that,
i) Points in the same direction as $\mathbf{v}$
ii) Points in the opposite direction as $\mathbf{v}$
d) Find the angle between the vectors $p=2 i+3 j+4 k$ and $q=4 i-3 j+2 k$
e) Determine if the following sets of vectors will span $\mathrm{R}^{3}$
i) $\quad \mathrm{V}_{1}=(2,0,1), \mathrm{v}_{2}=(-1,3,4)$ and $\mathrm{v}_{3}=(1,1,-2)$
ii) $\quad \mathrm{V} 2=(1,2,-1)$, v2 $=(3,-1,1)$ and $\mathrm{v} 3=(-3,8,-5)$

## QUESTION TWO (20 MARKS)

a) Given $\mathrm{u}=(3,-1,4)$ and $\mathrm{v}=(2,0,1)$ compute each of the following
i) $\quad u x v$ and $v x u$
(6 marks)
ii) $\quad \mathrm{uxu}^{\mathrm{u}}$
iii) $u .(u \times v)$ and $v .(u \times v)$
iv) Angle between $u$ and $v$
b) Find the rank of the following matrix

$$
\left(\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6 \\
2 & 1 & 0
\end{array}\right)
$$

## QUESTION THREE (20 MARKS)

a) Given $u=(-2,3,1,-1)$ and $v=(7,1,-4,-2)$ verify the cauchy-schwarz inequality and the triangular inequality
(6 marks)
b) Determine if the following sets of vectors are linearly independent or linearly dependent

> i) $\mathrm{V}_{1}=(1,1,-1,2), \mathrm{v}_{2}=(2,-2,0,2)$ and $\mathrm{v}_{3}=(2,-8,3,-1)$ ii) $\mathrm{v}_{1}{ }^{`}=(1,-2,3,-4), \mathrm{v}_{2}=(-1,3,4,2)$ and $\mathrm{v}_{3}=(1,1,-2,-2)$
c) Show that $\left(\mathrm{A}^{\mathrm{n}}\right)\left(\mathrm{A}^{-1}\right)=\mathrm{I}$
(4 marks)

## QUESTION FOUR (20 MARKS)

a) Suppose that $\mathbf{u}=\left(u_{1}, u_{2}, u_{3}\right)$ and $\mathbf{v}=\left(v_{1}, v_{2}, v_{3}\right)$ are two vectors in 3- space then,

Show that $u . v=u_{1} v_{1}+u_{2} v_{2}+u_{3} v_{3}$
(7 marks)
b) Solve the following systems of equations using cramers rule
$4 x+2 y+4 z=16$
$2 x-6 y+6 z=-8$
(7 marks)
$8 x+4 y-2 z=2$
c) Evaluate each of the following if $\mathrm{A}=\left(\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right)$
i) $\quad \mathrm{A}^{2}(3$ marks $)$
ii) $A^{3}$
(3 marks)

## QUESTION FIVE (20 MARKS)

a) For the following matrices compute

$$
A=\left(\begin{array}{cc}
10 & -6 \\
-3 & -1
\end{array}\right) \quad B=\left(\begin{array}{cc}
1 & 2 \\
5 & -6
\end{array}\right)
$$

i) $\quad \operatorname{Det}(\mathrm{A})$
ii) $\operatorname{Det}(\mathrm{B})$
iii) $\operatorname{Det}(A B)$
iv) $\quad \operatorname{Det} A^{-1}$
b) Show that $\mathrm{u}=(3,0,1,0,4,-1)$ and $\mathrm{v}=(-2,5,0,2,-3,-18)$ are orthogonal and verify that the pythagorean Theorem holds

