KABARAK UNIVERSITY

SUPPLEMENTARY/SPECIAL EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 211

COURSE TITLE: LINEAR ALGEBRA I

STREAM: SESSION II

DAY: **TUESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **17/03/2009**

INSTRUCTIONS TO CANDIDATES:

Answer questions **ONE (compulsory)** and any other **TWO**.

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Explain what is meant by a set of vectors being linear dependent. (3 mks)

(b) Show whether the following vectors form a basis for

R^3, $V_1 = (3, 1, -4)$

$V_2 = (2, 5, 6)$

$V_3 = (1, 4, 8)$ (8 mks)

(c) Derive the triangle inequality

$$|u+v| \subseteq |u| + |v|$$ (8 mks)

(d) Find the angle between ***U***= (-1, 3, 4) and ***V*** = (-2 . -1, 3) (3 mks)

(e) Consider the matrix equation

$$\begin{bmatrix} -1 & 3 & 2 \\ 0 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix}$$

For what values of b does a solution exist? What is the solution? (8mks)

QUESTION TWO (20 MARKS)

(a) (i) Given H and K as vector subspaces of a vector space V, show that H f K is a vector subspace of V. (4mks)

(ii) Let H = $\{(x, y, z) \mid 2x - y - z = 0\}$ and K = $\{(x, y, z) \mid x + 2y + 3z = 0\}$ be vector subspaces of R^3. Find H f K and determine whether it is a subspace of R^3. (7mks)

(b) (i) Let V = $\{v_1, v_2, —, v_n\}$ be a vector space in R^3. Explain clearly what is meant by V is linearly dependent. (2mks)

(ii) Determine whether or not the set V = $\{(1,0,2,3), (4,0,5,6), (7,0,8,9)\}$ in R^4 is linearly independent. If not, find a linear combination that equals 0. (zero). (7mks)

QUESTION THREE (20 marks)

(a) Find the general solution of the homogenous system of linear equations given below:-

$x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 0$
$x_1 - x_2 - x_3 + x_4 - x_5 = 0$
$2x_1 + x_3 + x_5 = 0$
$3x_1 + x_2 + x_4 - 3x_5 = 0$

(10 marks)

(b) Use Cramer's Rule to solve the non-homogenous system of linear equations;

$2y + 2x = z + 1$
$3x + 2z = 8 - 5y$
$3z - 1 = x - 2y$ (10 marks)

QUESTION FOUR (20 MARKS)

(a) Let V and W be two vector spaces. Define a linear transformation from V into W. (3mks)

(b) A linear transformation T: $R^3 \to R^3$ is defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x & -y & +3z \\ 4x & -3y & +6z \\ -6x & +3y & -9z \end{bmatrix}$$

Determine

(i) the standard matrix A that T represents (1mk)
(ii) the rank of A (2mks)
(iii) the range of A (3mks)
(iv) the kernel of A (3mks)
(v) the nullity of A (2mks)

(c) Let T: $R^3 \to R^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$$

Find

(i) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$ (4mks)

(ii) $T\left(\begin{bmatrix} -3 \\ 7 \end{bmatrix}\right)$ (2mks)

QUESTION FIVE (20 MARKS)

(a) (i) If u and v and two non-zero vectors in 3-space, then derive the orthogonal projection of u on v and the component of u orthogonal to v. (3 marks)

(ii) If u = (3,-4,2) and v = (2,1,-2) are two vectors in 3-space then find the orthogonal projection of u on v and the component of u orthogonal to v. (8 marks)

(iii) From your results in (ii) confirm that w_2 is perpendicular to v. (2 marks)

(b) From the vectors u and v given in (a) (ii), find u x v (1 marks)

(c) Show that the cross product u x v is orthogonal to each vector u and v. (2 marks)

(d) If u, v, and w are vectors in the vector space R^3, then show that ;
(u + v) x w = (u x w) + (v x w) . (4 marks)