KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 211

COURSE TITLE: LINEAR ALGEBRA ONE

STREAM: SESSION IV

DAY: **SATURDAY**

TIME: **11.00 – 1.00 P.M.**

DATE: **29/11/08**

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Find the determinants of the following matrices:

(i) $\begin{bmatrix} 6 & -4 \\ -3 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 3 \\ 2 & 1 & -4 \end{bmatrix}$ **(5 mks)**

(b) Solve the following system of equations using row reduction method

$x + y - z = 7$

$4x - y - 5z = 4$

$2x + 2y - 3z = 0$

(5 mks)

(c) Consider the matrix

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix}$$

i) Find the nullspace of A:

ii) Write the general solution of the equation

$Ax = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$ in the form of a particular solution plus an arbitrary member of the nullspce of A. **(6 mks)**

(d) (i) Determine the angle between the vectors

$a = 2i - j + 6k$ and $b = 3i + 2j + k$ **(4 mks)**

(ii) L et $T: \Re^2 \rightarrow \Re^3$ be linear transformation such that

$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$, find $\begin{bmatrix} x \\ y \end{bmatrix}$ **(4 mks)**

(e) (i) Given $\underline{a} = 3\underline{i} + 2\underline{j} + 4\underline{k}$ and $\underline{b} = -2\underline{i} + 4\underline{j} - \underline{k}$, determine $\underline{a} \times \underline{b}$ **(3 mks)**

(ii) Determine the value of λ for which the vectors a = 4λ i + λ j – k and

b = λ i + 2j + 2k are orthogonal **(3 mks)**

QUESTION TWO (20 MARKS)

(a) State and prove the Schwarz inequality

(10 mks)

(b) The planes x + 2y + 3z = 2 and 2x +3y + 2z = 4 intersect along line L. Find the parametric equation of L and state its direction vector **(5 mks)**

(c) Assume that P is a particular solution to the matrix equation Ax = b and let N be the nullspace of A. Show that the entire solution set to this equation is (P + n: n EN). **(5 mks)**

QUESTION THREE (20 MKS)

(a) Consider a line whose equation is y = 3x + 2.

Investigate if this line is a vector space **(7 mks)**

(b) Let H = $\{(x, y, z)\}$ 2x – y - z = 0$\}$ and of K = $\{(x, y, z)/\ x + 2y + 3z = 0\}$ be subspaces of $\Re^3$. Find H f K and determine whether it is a subspace of $\Re^3$ **(10 mks)**

(c) Define the term nullspace. **(3 mks)**

QUESTION FOUR (20 MKS)

(a) Given $V_1 = (1, 4, 6)$, $V_2 = (1, -1, 1)$ and $V_3 = (-1, 5, 3)$,
Determine if (7, 4, 2) is in Sp (v_1, v_2, v_3) **(5 mks)**

(b) A linear mapping T: $\Re^3 \rightarrow \Re^3$ is defined by

$$T(|x, y, z|) = [2x + 3z, 3y + 2z, 2x + 5y]$$

(i) Find a matrix M_T that represents T with respect to the standard ordered basis for $\Re^3$

(3 mks)

(ii) Define the vector a if a is in the Kernel of T. **(5 mks)**

(iii) Determine the vectors which span the range of T. **(5 mks)**

(iv) Determine the nullity of M_T **(2 mks)**

QUESTION FIVE (20 MARKS)

(a) When are vectors V_1, V_2, ----V_n said to be linearly dependent? **(3 mks)**

(b) Consider the subspace S of $\Re^3$ given by S = sp $\{(2,1,3),(-1,2,0),(1,8,6)\}$

Use the casting technique to find a basis for S **(12 mks)**

(c) Find the dimension of the nullspace of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

(5 mks)