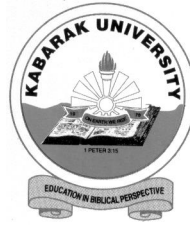


KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE**

COURSE CODE: MATH 211

COURSE TITLE: LINEAR ALGEBRA ONE

STREAM: SESSION IV

DAY: SATURDAY

TIME: 11.00 – 1.00 P.M.

DATE: 29/11/08

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Find the determinants of the following matrices:

$$(i) \begin{bmatrix} 6 & -4 \\ -3 & 2 \end{bmatrix} \quad (ii) \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 3 \\ 2 & 1 & -4 \end{bmatrix} \quad (5 \text{ mks})$$

(b) Solve the following system of equations using row reduction method

$$x + y - z = 7$$

$$4x - y - 5z = 4$$

$$2x + 2y - 3z = 0$$

(5 mks)

(c) Consider the matrix

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix}$$

i) Find the nullspace of A:

ii) Write the general solution of the equation

$$Ax = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} \text{ in the form of a particular solution plus an arbitrary member of the nullspace of A.} \quad (6 \text{ mks})$$

(d) (i) Determine the angle between the vectors

$$a = 2i - j + 6k \text{ and } b = 3i + 2j + k \quad (4 \text{ mks})$$

(ii) Let $T: \mathcal{R}^2 \rightarrow \mathcal{R}^3$ be linear transformation such that

$$T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, T \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}, \text{ find } \begin{bmatrix} x \\ y \end{bmatrix} \quad (4 \text{ mks})$$

(e) (i) Given $\underline{a} = 3\underline{i} + 2\underline{j} + 4\underline{k}$ and $\underline{b} = -2\underline{i} + 4\underline{j} - \underline{k}$, determine $\underline{a} \times \underline{b}$ (3 mks)

- (ii) Determine the value of λ for which the vectors $a = 4\lambda i + \lambda j - k$ and $b = \lambda i + 2j + 2k$ are orthogonal (3 mks)

QUESTION TWO (20 MARKS)

- (a) State and prove the Schwarz inequality (10 mks)
- (b) The planes $x + 2y + 3z = 2$ and $2x + 3y + 2z = 4$ intersect along line L. Find the parametric equation of L and state its direction vector (5 mks)
- (c) Assume that P is a particular solution to the matrix equation $Ax = b$ and let N be the nullspace of A. Show that the entire solution set to this equation is $(P + n: n \in N)$. (5 mks)

QUESTION THREE (20 MKS)

- (a) Consider a line whose equation is $y = 3x + 2$. Investigate if this line is a vector space (7 mks)
- (b) Let $H = \{(x, y, z) \mid 2x - y - z = 0\}$ and of $K = \{(x, y, z) \mid x + 2y + 3z = 0\}$ be subspaces of \mathfrak{R}^3 . Find $H \cap K$ and determine whether it is a subspace of \mathfrak{R}^3 (10 mks)
- (c) Define the term nullspace. (3 mks)

QUESTION FOUR (20 MKS)

- (a) Given $V_1 = (1, 4, 6)$, $V_2 = (1, -1, 1)$ and $V_3 = (-1, 5, 3)$, Determine if $(7, 4, 2)$ is in $\text{Sp}(v_1, v_2, v_3)$ (5 mks)
- (b) A linear mapping $T: \mathfrak{R}^3 \rightarrow \mathfrak{R}^3$ is defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x + 3z \\ 3y + 2z \\ 2x + 5y \end{bmatrix}$
- (i) Find a matrix M_T that represents T with respect to the standard ordered basis for \mathfrak{R}^3 (3 mks)
- (ii) Define the vector a if a is in the Kernel of T. (5 mks)
- (iii) Determine the vectors which span the range of T. (5 mks)

(iv) Determine the nullity of M_T **(2 mks)**

QUESTION FIVE (20 MARKS)

(a) When are vectors V_1, V_2, \dots, V_n said to be linearly dependent? **(3 mks)**

(b) Consider the subspace S of \mathcal{R}^3 given by $S = \text{sp}\{(2,1,3), (-1,2,0), (1,8,6)\}$

Use the casting technique to find a basis for S **(12 mks)**

(c) Find the dimension of the nullspace of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

(5 mks)